Meeting 1: Introduction, Nuclear Properties, Decay Kinetics, Alpha Decay

- Readings:
  - Chart of the nuclides
    → Class handout
  - Table of the isotopes
  - Modern Nuclear Chemistry: Chapter 1
    → http://radchem.nevada.edu/docs/course%20reading/Nuc%20&%20Radchem%203rd%20Ed%20Friedlander.pdf

- Class organization
- Outcomes
- Grading
- Resources
  - Chart of the nuclides book (bring to class everyday!)
  - Electronic resources
    → Web pages, pdfs, apps, programs, blog
- History of radiation research
- Chart of the nuclides and Table of the isotopes
  - Description and use
  - Data
- Radiochemistry introduction
  - Atomic properties
  - Nuclear nomenclature
  - X-rays
  - Types of decays
  - Forces (limit of course instruction)
Course overview

• Radiochemistry includes physics of radioactive decay and chemistry of radioisotopes
  ▪ Intellectual intersection of the periodic table and chart of the nuclides
    → Emphasis on elements with only radioactive isotopes
      * Tc, actinides
• Course topics
  ▪ Chart of the nuclides
  ▪ Details on alpha decay, beta decay, gamma decay, and fission
  ▪ Methods and data from the investigation of nuclear properties
  ▪ Fundamental chemical properties in radiation and radiochemistry
  ▪ Radioisotope production and
  ▪ Radiochemistry in research and technology
• Textbooks and published literature are used as reading material
  ▪ Available as PDFs
    → Linked to web page
• Input from students valued
  ▪ Expect participation and assistance with course development
  ▪ Output should enhance on-line course
Outcomes

1. Understand, utilize, and apply the chart of the nuclides and table of the isotopes to radiochemistry and nuclear technology
2. Understand the fundamentals of nuclear structure
3. Understand chemical properties of radioelements
4. Comprehend and evaluate nuclear reactions and the production of isotopes
5. Comprehend types and descriptions of radioactive decay
6. Utilization of radiochemistry in research
7. Evaluation of concentration
8. Behavior of radioelements
9. Exploitation of isotopes
10. Investigate modern topics relating radiochemistry to the nuclear fuel cycle
History of Radiation Research

- 1896  Discovery of radioactivity
  - Becquerel used $\text{K}_4\text{UO}_2(\text{SO}_4)_2\cdot\text{H}_2\text{O}$ exposed to sunlight and placed on photographic plates wrapped in black paper
- 1898  Isolation of radium and polonium
- 1899  Radiation into alpha, beta, and gamma components, based on penetration of objects and ability to cause ionization
- 1909  Alpha particle shown to be He nucleus
  - Charge to mass determined by Rutherford
- 1911  Nuclear atom model
  - Plum pudding by Thomson examined
  - Rutherford developed planetary model
- 1912  Development of cloud chamber by Wilson
- 1913  Planetary atomic model expanded (Bohr Model)
  - Application of quantum mechanics
- 1914  Nuclear charge determined from X rays
  - Determined by Moseley in Rutherford’s laboratory
- 1919  Artificial transmutation by nuclear reactions
  - Rutherford bombarded $^{14}\text{N}$ with alpha particle to make $^{17}\text{O}$
- 1919  Development of mass spectrometer
- 1928  Theory of alpha radioactivity
  - Tunneling description by Gamow
- 1930  Neutrino hypothesis
  - Fermi, mass less particle with $\frac{1}{2}$ spin, explains beta decay
- 1932  First cyclotron
  - Lawrence at UC Berkeley
- 1932  Discovery of neutron
- 1934  Discovery of artificial radioactivity
- 1938  Discovery of nuclear fission
  - From reaction of U with neutrons, Hahn and Meitner
- 1942  First controlled fission reactor
  - Chicago Pile
- 1945  First fission bomb tested
  - Trinity Test
- 1947  Development of radiocarbon dating
Radioelements

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*Lanthanides*

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Technetium

- Confirmed in a December 1936 experiment at the University of Palermo
  - Carlo Perrier and Emilio Segrè.
  - Ernest Lawrence (UC Berkeley) mailed molybdenum foil from cyclotron deflector
  - Succeeded in isolating the isotopes $^{95,97}\text{Tc}$
  - Named after Greek word τεχνητός, meaning artificial
    ➔ University of Palermo officials wanted them to name their discovery "panormium", after the Latin name for Palermo, *Panormus*
  - Segre and Seaborg isolate $^{99m}\text{Tc}$
Promethium

• Promethium was first produced and characterized at ORNL in 1945 by Jacob A. Marinsky, Lawrence E. Glendenin and Charles D. Coryell

• Separation and analysis of the fission products of uranium fuel irradiated in the Graphite Reactor

• Announced discovery in 1947

• In 1963, ion-exchange methods were used at ORNL to prepare about 10 grams of Pm from used nuclear fuel
Np synthesis

- Neptunium was the first synthetic transuranium element of the actinide series discovered
  - Isotope $^{239}\text{Np}$ was produced by McMillan and Abelson in 1940 at Berkeley, California
  - Bombarding uranium with cyclotron-produced neutrons
    $\rightarrow 238\text{U}(n,\gamma)239\text{U}$, beta decay of $239\text{U}$ to $239\text{Np}$ ($t_{1/2}=2.36 \text{ days}$)
  - Chemical properties unclear at time of discovery
    $\rightarrow$ Actinide elements not in current location
    $\rightarrow$ In group with W
- Chemical studies showed similar properties to U
- First evidence of 5f shell
- Macroscopic amounts
  - $^{237}\text{Np}$
    $\rightarrow 238\text{U}(n,2n)237\text{U}$
    * Beta decay of $237\text{U}$
    $\rightarrow 10\text{ microgram}$
Pu synthesis

- Plutonium was the second transuranium element of the actinide series to be discovered
  - The isotope $^{238}\text{Pu}$ was produced in 1940 by Seaborg, McMillan, Kennedy, and Wahl
  - Deuteron bombardment of U in the 60-inch cyclotron at Berkeley, California
    \[ {^{238}\text{U}}(^{2}\text{H},\ 2\text{n})^{238}\text{Np} \]
    * Beta decay of $^{238}\text{Np}$ to $^{238}\text{Pu}$
  - Oxidation of produced Pu showed chemically different
- $^{239}\text{Pu}$ produced in 1941
  - Uranyl nitrate in paraffin block behind Be target bombarded with deuterium
  - Separation with fluorides and extraction with diethylether
  - Eventually showed isotope undergoes slow neutron fission
Am and Cm discovery

• First produce in reactor via neutron capture
  ▪ neutron capture on $^{239}\text{Pu}$
  ▪ $^{239}\text{Pu} + n \rightarrow ^{240}\text{Pu} + n \rightarrow ^{241}\text{Pu} \rightarrow ^{241}\text{Am} + \beta^-$
  ▪ Also formed $^{242}\text{Cm}$

• Direct production
  ▪ $^{241}\text{Am}$ from $^{241}\text{Pu}$ produced by $^{238}\text{U} + ^4\text{He}$
    → Also directly produced from He on $^{237}\text{Np}$ and $^2\text{H}$ on $^{239}\text{Pu}$
  ▪ $^{239}\text{Pu}(^4\text{He},n)^{242}\text{Cm}$
    → Chemical separation from Pu
    → Identification of $^{238}\text{Pu}$ daughter from alpha decay

• Difficulties in separating Am from Cm and from lanthanide fission products
  ▪ Trivalent oxidation states

• See publications announcing discovery on web page
Bk and Cf discovery

- Required Am and Cm as targets
  - Needed to produce theses isotopes in sufficient quantities → Milligrams
  - Am from neutron reaction with Pu
  - Cm from neutron reaction with Am
- Production of new elements followed by separation
  - $^{241}\text{Am}(^{4}\text{He},2n)^{243}\text{Bk}$ → Cation exchange separation
  - $^{242}\text{Cm}(^{4}\text{He},n)^{245}\text{Cf}$ → Anion exchange
- Where would the heavier actinides elute?

Dowex 50 resin at 87 °C, elute with ammonium citrate
Einsteinium and Fermium

- Debris from Mike test
  - 1st thermonuclear test
    → [http://www.youtube.com/watch?v=h7vyKDcS TaE](http://www.youtube.com/watch?v=h7vyKDcS TaE)

- New isotopes of Pu
  - 244 and 246
    → Successive neutron capture of $^{238}\text{U}$
  - Correlation of log yield versus atomic mass

- Evidence for production of transcalifornium isotopes
  - Heavy U isotopes followed by beta decay
  - Successive neutron capture to form Es and Fm
    → Similar to r-process in nucleosynthesis

- Ion exchange used to separate new elements
Md, No, and Lr discovery

- 1st atom-at-a-time chemistry
  - $^{253}\text{Es}(^4\text{He},n)^{256}\text{Md}$
- Required high degree of chemical separation
- Use catcher foil
  - Recoil of product onto foil
  - Dissolved Au foil, then ion exchange
- Nobelium controversy
  - Expected to have trivalent chemistry
    - Actually divalent, filled 5f orbital
      - Divalent from removing 7s electrons
  - 1st attempt could not be reproduced
    - Showed divalent oxidation state
  - $^{246}\text{Cm}(^{12}\text{C},4n)^{254}\text{No}$
    - Alpha decay from $^{254}\text{No}$
    - Identification of $^{250}\text{Fm}$ daughter using ion exchange
- For Lr $^{249, 250, 251}\text{Cf}$ bombarded with $^{10, 11}\text{B}$
- New isotope with 8.6 MeV, 6 second half life
  - Identified at $^{258}\text{Lr}$
Radiochemistry terms and concepts

- **Radiochemistry**
  - Chemistry of the radioactive isotopes and elements
  - Utilization of nuclear properties in evaluating and understanding chemistry
  - Intersection of chart of the nuclides and periodic table

- **Atom**
  - **Z and N in nucleus (10^{-14} \text{ m})**
  - Electron interaction with nucleus basis of chemical properties (10^{-10} \text{ m})
    - Electrons can be excited
      - Higher energy orbitals
      - Ionization
        - Binding energy of electron effects ionization
  - **Isotopes**
    - Same Z different N
  - **Isobar**
    - Same A (sum of Z and N)
  - **Isotone**
    - Same N, different Z
  - **Isomer**
    - Nuclide in excited state
    - $^{99m}_{\text{ChemicalSymbol}}{\text{Tc}}$
Types of Decay

1. $\alpha$ decay (occurs among the heavier elements)
$$^{226}_{88}Ra \rightarrow ^{222}_{86}Rn + ^4_2\alpha + \text{Energy}$$

2. $\beta^-$ decay
$$^{131}_{53}I \rightarrow ^{131}_{54}Xe + \beta^- + \bar{\nu} + \text{Energy}$$

3. Positron emission
$$^{22}_{11}Na \rightarrow ^{22}_{10}Ne + \beta^+ + \nu + \text{Energy}$$

4. Electron capture
$$^{26}_{13}Al + \beta^- \rightarrow ^{26}_{12}Mg + \nu + \text{Energy}$$

5. Spontaneous fission
$$^{252}_{98}Cf \rightarrow ^{140}_{54}Xe + ^{108}_{44}Ru + 4\cdot^1_0n + \text{Energy}$$
Fission Products

- Fission yield curve varies with fissile isotope
- 2 peak areas for U and Pu thermal neutron induced fission
- Variation in light fragment peak
- Influence of neutron energy observed

![Diagram showing fission yield for U-235 and Pu-239]
Photon emission

- **Gamma decay**
  - Emission of photon from excited nucleus
    - Metastable nuclide (i.e., $^{99m}\text{Tc}$)
    - Following decay to excited daughter state

- **X-ray**
  - Electron from a lower level is removed
    - Electrons from higher levels occupy resulting vacancy with photon emission
  - De-acceleration of high energy electrons
  - Electron transitions from inner orbitals
  - X-ray production
    - Bombardment of metal with high energy electrons
    - Secondary x-ray fluorescence by primary x-rays
    - Radioactive sources
    - Synchrotron sources

![Gamma Decay Diagram](gamma_decay.png)

![X-ray Diagram](x-ray_diagram.png)

Figure 4 Moseley relationship for $K\alpha$ and $L\alpha$ radiation
Chart of the Nuclides

- Presentation of data on nuclides
  - Information on chemical element
  - Nuclide information
    - Spin and parity (0^+ for even-even nuclides)
    - Fission yield
  - Stable isotope
    - Isotopic abundance
    - Reaction cross sections
    - Mass
- Radioactive isotope
  - Half-life
  - Modes of decay and energies
  - Beta disintegration energies
  - Isomeric states
  - Natural decay series
  - Reaction cross sections
- Fission yields for isobars
Chart of the nuclides

Member of Naturally Radioactive Decay Chain

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Mode of decay in order of prominence with energy of radiation in MeV for alpha and beta, keV for gammas

- Thermal neutron capture cross section

Two Isomeric States One Stable

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Spin and parity of metastable state, 11/2$^-$

Mode of decay in order of prominence with energy of radiation in MeV for alpha and beta, keV for gammas

- Thermal neutron capture cross section in barns, followed by resonance integral in barns
- Fission product from the slow neutron fission of U235
- Stable ground state isomer

Two Isomeric States Both Radioactive

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Spin and parity of metastable state, 11/2$^-$

Mode of decay in order of prominence with energy of radiation in MeV for alpha and beta, keV for gammas

- Beta disintegration energy in MeV
- Fission product from the slow neutron fission of U235
- Radioactive ground state isomer
Chart of the Nuclide: Fission yields
Chart of the Nuclides Questions

- How many stable isotopes of Ni?
- What is the mass and isotopic abundance of $^{84}$Sr?
- Spin and parity of $^{201}$Hg?
- Decay modes and decay energies of $^{212}$Bi
- What are the isotopes in the $^{235}$U decay series?
- What is the half-life of $^{176}$Lu?
- What is the half-life of $^{176}$Yb
- How is $^{238}$Pu produced?
- How is $^{239}$Pu made from $^{238}$U
- Which actinide isotopes are likely to undergo neutron induced fission?
- Which isotopes are likely to undergo alpha decay?

- What is the half life of $^{130}$Te
  - What is its decay mode?
- What cross section data is provided for $^{130}$Te?
Table of the Isotopes

- Detailed information about each isotope
  - Mass chain decay scheme
  - Mass excess (M-A)
    - Mass difference, units in energy (MeV)
  - Particle separation energy
  - Populating reactions and decay modes
  - Gamma data
    - Transitions, % intensities
  - Decay levels
    - Energy, spin, parity, half-life
  - Structure drawing
- Example with $^{99}$Mo
  - Show isobar, $^{99}$Mo data, gamma decay energy, level scheme to $^{99m}$Tc decay
Half Lives

\[
\frac{N}{N_0} = e^{-\lambda t}
\]

\[
N = N_0 e^{-\lambda t}
\]

\[
\lambda = \frac{\ln 2}{t_{1/2}}
\]

\(\lambda\) is decay constant

\(N_0\) = number at time zero (atoms, mass, moles)

\(N\) = number at time \(t\)

Rate of decay of \(^{131}\text{I}\) as a function of time.
Lecture 1: Topic review

- History of nuclear physics research
- Discovery of the radioelements
  - Methods and techniques used
- Types of radioactive decay
  - Define X-rays and gamma decay
- Understand and utilize the data presented in the chart of the nuclides and table of the isotopes
- Utilize the fundamental decay equations
- Identify common fission products
Lecture 1: Study Questions

- What are the course outcomes?
- What were important historical moments in radiochemistry?
- Who were the important scientists in the investigation of nuclear properties?
- What are the different types of radioactive decay?
- What are some commonalities in the discovery of the actinides?
- Provide 5 radioelements
- Pop Quiz: Provide 10 facts about $^{239}$Pu using the chart of the nuclide or the table of the isotopes
RFSS: Lecture 2
Nuclear Properties

- **Readings:**
  - Modern Nuclear Chemistry: Chapter 2 Nuclear Properties
  - Nuclear and Radiochemistry Chapter 1 Introduction, Chapter 2 Atomic Nuclei

- **Nuclear properties**
  - Masses
  - Binding energies
  - Reaction energetics
    - \( Q \) value
  - Nuclei have shapes

Simple example: Number of stable nuclei based on neutron and proton number

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Simple property dictates nucleus behavior. Number of protons and neutron important.
Data from Mass

- Evaluation of Mass Excess
- Difference between actual mass of nucleus and expected mass from atomic number
  - By definition $^{12}\text{C} = 12 \text{ amu}$
    - If mass excess negative, then isotope has more binding energy than $^{12}\text{C}$
- Mass excess $= \Delta = M - A$
  - $M$ is nuclear mass, $A$ is mass number
  - Unit is MeV (energy)
    - Convert with $E = mc^2$
- $^{24}\text{Na}$ example
  - $23.990962782$ amu
  - $23.990962782 - 24 = -0.009037218$ amu
  - $1 \text{ amu} = 931.5 \text{ MeV}$
    - $-0.009037218 \text{ amu} \times (931.5 \text{ MeV/1 amu})$
    - $-8.41817 \text{ MeV} = \text{Mass excess} = \Delta$ for $^{24}\text{Na}$
- Question: What is the relative small between an electron and a nucleon?
Masses and Q value

- **Atomic masses**
  - From nuclei and electrons

- **Nuclear mass can be found from atomic mass**
  \[
  m_{\text{nucl}} c^2 = M_{\text{atomic}} c^2 - \left[Z m_0 c^2 + B_e(Z)\right]
  \]
  - \(m_0\) is electron rest mass, \(B_e(Z)\) is the total binding energy of all the electrons
  \[
  B_e(Z) = 15.73 Z^{7/3} \text{ eV}
  \]
  - \(B_e(Z)\) is small compared to total mass

- **Energy (Q) from mass difference between parent and daughter**
  - Mass excess values can be used to find Q (in MeV)

- **\(\beta^-\) decay Q value**
  - \(^{14}\text{C} \rightarrow ^{14}\text{N}^+ + \beta^- + \bar{\nu} + Q\)
    - Consider \(\beta^-\) mass to be part of \(^{14}(Z+1)\) atomic mass (neglect binding)
    - \(Q = \Delta ^{14}Z - \Delta ^{14}(Z+1)\)
  - \(^{14}\text{C} \rightarrow ^{14}\text{N}^+ + \beta^- + \bar{\nu} + Q\)
    - Energy = \(Q = \text{mass } ^{14}\text{C} - \text{mass } ^{14}\text{N}\)
      - Use Q values
        \(\text{(http://www.nndc.bnl.gov/wallet/wccurrent.html)}\)
    - \(Q = 3.0198 - 2.8634 = 0.156\text{ MeV}\)
**Q value**

- **Positron Decay**
  - \(^{A}Z\rightarrow^{A}(Z-1)^{-} + \beta^{+} + \nu + Q\)
  - Have 2 extra electrons to consider
    - \(\rightarrow \beta^{+} \) (positron) and additional atomic electron from \(Z-1\) daughter
      * Each electron mass is 0.511 MeV, 1.022 MeV total from the electrons
  - \(Q=\Delta^{A}Z – (\Delta^{A}(Z-1)^{-} + 1.022) \) MeV

- **Electron Capture (EC)**
  - Electron comes from parent orbital
    - \(\rightarrow \) Parent can be designated as cation to represent this behavior
  - \(^{A}Z^{+} + e^{-} \rightarrow^{A}(Z-1) + \nu + Q\)
  - \(Q=\Delta^{A}Z – \Delta^{A}(Z-1)\)

- **Alpha Decay**
  - \(^{A}Z\rightarrow^{(A-4)}(Z-2) + ^{4}\text{He} + Q\)
  - \(^{241}\text{Am} \rightarrow^{237}\text{Np} + ^{4}\text{He} + Q\)
Q value determination

• For a general reaction
  ▪ Treat Energy (Q) as part of the equation
    → Solve for Q
  • 56\text{Fe} + 4\text{He} \rightarrow 59\text{Co} + 1\text{H} + Q
    ▪ Q = [M^{56}\text{Fe} + M^4\text{He} - (M^{59}\text{Co} + M^1\text{H})]c^2
    * M represents mass of isotope
    → Q = -3.241 \text{ MeV} (from Q value calculator)

• Mass excess and Q value data can be found in a number of sources
  ▪ Table of the Isotopes
  ▪ Q value calculator
  ▪ Atomic masses of isotopes
    → http://physics.nist.gov/cgi-bin/Compositions/stand_alone.pl

• Q value examples
• Turn 208\text{Pb} in Au with a proton
• 261\text{Rf} from the reaction of 18\text{O} and 248\text{Cm}
• Alpha decay of 208\text{Pb}
• Compare neutron capture on 239\text{Pu} and 240\text{Pu}
Terms from Energy

• **Binding energy**
  - Difference between mass of nucleus and constituent nucleons
    - Energy released if nucleons formed nucleus
  - Nuclear mass not equal to sum of constituent nucleons
    \[ B_{\text{tot}}(A,Z) = [ZM(^1\text{H}) + (A-Z)M(n) - M(A,Z)]c^2 \]
    - average binding energy per nucleon
      \[ B_{\text{ave}}(A,Z) = \frac{B_{\text{tot}}(A,Z)}{A} \]
      - Some mass converted into energy that binds nucleus
      - Measures relative stability

• **Binding Energy of an even-A nucleus is generally higher than adjacent odd-A nuclei**
• **Exothermic fusion of H atoms to form He from very large binding energy of \(^4\text{He}\)**
• **Energy released from fission of the heaviest nuclei is large**
  - Nuclei near the middle of the periodic table have higher binding energies per nucleon
• **Maximum in the nuclear stability curve in the iron-nickel region (A~56 through 59)**
  - Responsible for the abnormally high natural abundances of these elements
  - Elements up to Fe formed in stellar fusion
Binding-Energy Calculation: Development of simple nuclear model

- Volume of nuclei are nearly proportional to number of nucleons present
  - Nuclear matter is incompressible
  - Basis of equation for nuclear radius
- Total binding energies of nuclei are nearly proportional to numbers of nucleons present
  - saturation character
    - Nucleon in a nucleus can apparently interact with only a small number of other nucleons
    - Those nucleons on the surface will have different interactions
- Basis of liquid-drop model of nucleus
  - Considers number of neutrons and protons in nucleus and how they may arrange
  - Developed from mass data
Liquid-Drop Binding Energy:

\[ E_B = c_1 A \left[ 1 - k \left( \frac{N - Z}{A} \right)^2 \right] - c_2 A^{2/3} \left[ 1 - k \left( \frac{N - Z}{A} \right)^2 \right] - c_3 Z^2 A^{-1/3} + c_4 Z^2 A^{-1} + \delta \]

- \( c_1 = 15.677 \text{ MeV}, c_2 = 18.56 \text{ MeV}, c_3 = 0.717 \text{ MeV}, c_4 = 1.211 \text{ MeV}, k = 1.79 \) and \( \delta = 11/A^{1/2} \)

- 1st Term: Volume Energy
  - dominant term
    - in first approximation, binding energy is proportional to the number of nucleons
  - \((N-Z)^2/A\) represents symmetry energy
    - binding \( E \) due to nuclear forces is greatest for the nucleus with equal numbers of neutrons and protons
Liquid drop model

\[ E_B = c_1 A \left[ 1 - k \left( \frac{N - Z}{A} \right)^2 \right] - c_2 A^{2/3} \left[ 1 - k \left( \frac{N - Z}{A} \right)^2 \right] - c_3 Z^2 A^{-1/3} + c_4 Z^2 A^{-1} + \delta \]

- **2nd Term: Surface Energy**
  - Nucleons at surface of nucleus have unsaturated forces
  - decreasing importance with increasing nuclear size

- **3rd and 4th Terms: Coulomb Energy**
  - 3rd term represents the electrostatic energy that arises from the Coulomb repulsion between the protons → lowers binding energy
  - 4th term represents correction term for charge distribution with diffuse boundary

- **\( \delta \) term: Pairing Energy**
  - binding energies for a given \( A \) depend on whether \( N \) and \( Z \) are even or odd
    - even-even nuclei, where \( \delta = 11/A^{1/2} \), are the most stable
  - two like particles tend to complete an energy level by pairing opposite spins → Neutron and proton pairs

---

Fig. 2-9 Differences between experimental and liquid-drop-formula masses. In top graph isotones, in bottom graph isotopes are connected by lines. (From reference M2; drawing made available by J. R. Nix.)
mass parabola

Fig. 2-7. Mass parabola for $A = 75$ and $A = 157$, as calculated from (2-8). Calculated mass differences between neighboring isotopes are indicated, with experimentally determined values shown in parentheses for comparison. The top Z scale refers to $A = 75$, the bottom one to $A = 157$. 

Evaluator: R.G. Helmer
Nuclear Force Radii

- Nuclear volumes are nearly proportional to nuclear masses
  - \( r_o \sim 1.1 \text{ to } 1.6 \text{ fm for equation} \)
- The radius of the nuclear force field must be less than the distance of closest approach (\( d_o \))
  - \( d = \) distance from center of nucleus
  - \( T' = \alpha \) particle’s kinetic energy
  - \( T = \alpha \) particle’s initial kinetic energy
  - \( d_o = \) distance of closest approach in a head on collision when \( T' = 0 \)
- \( d_o \sim 10-20 \text{ fm for Cu and 30-60 fm for U} \)

\[ R = r_o A^{1/3} \]

\[ T' = T - \frac{2Ze^2}{d_o} \]

\[ d_o = \frac{2Ze^2}{T} \]

http://hyperphysics.phy-astr.gsu.edu/hbase/rutsca.html#c1
Measurement of Nuclear Radii

• Any positively charged particle can be used to probe the distance
  - nuclear (attractive) forces become significant relative to the Coulombic (repulsive force)
• Neutrons can be used but require high energy
  - neutrons are not subject to Coulomb forces
    → high energy needed for de Broglie wavelengths small compared to nuclear dimensions
  - at high energies, nuclei become transparent to neutrons
    → Small cross sections
• Radii determine by electrons distinctly smaller than indicated by methods that determine nuclear force radii
  • $R_e$ (half-density radius)\(\sim\)1.07 fm
  • $d_e$ (“skin thickness”)\(\sim\)2.4 fm
Nuclear potentials

- Scattering experimental data have approximate agreement with the Square-Well potential
- Woods-Saxon equation better fit

\[ V = \frac{V_o}{1 + e^{(r-R)/A}} \]

- \( V_o \) = potential at center of nucleus
- \( A = \text{constant} \approx 0.5 \text{ fm} \)
- \( R = \text{distance from center at which } V = 0.5V_o \) (for half-potential radii)
- or \( V = 0.9V_o \) and \( V = 0.1V_o \) for a drop-off from 90 to 10\% of the full potential

- \( r_o \approx 1.35 \) to 1.6 fm for Square-Well
- \( r_o \approx 1.25 \) fm for Woods-Saxon with half-potential radii,
- \( r_o \approx 2.2 \) fm for Woods-Saxon with drop-off from 90 to 10\%

- Nuclear skin thickness

Fig. 2-2 Potential energy as a function of distance from the center of a nucleus for (a) proton, (b) neutron. The solid curves represent square-well potentials, the dashed curves Woods-Saxon potentials. In (a) the dot-dash curve is the Coulomb potential \( V_c \) inside the nucleus.
Nuclear Skin

Nucleus Fraction of nucleons in the “skin”

\[
\begin{align*}
^{12}\text{C} & : 0.90 \\
^{24}\text{Mg} & : 0.79 \\
^{56}\text{Fe} & : 0.65 \\
^{107}\text{Ag} & : 0.55 \\
^{139}\text{Ba} & : 0.51 \\
^{208}\text{Pb} & : 0.46 \\
^{238}\text{U} & : 0.44
\end{align*}
\]

\[\rho(r) = \frac{\rho_o}{1 + e^{[(r_e - R_e) / a_e]}}\]
Magnetic methods of measurements

- Nuclei with nonzero angular momenta have magnetic moments
- Hyperfine structure in atomic spectra
- Atomic Beam method
  - Element beam split into 2I+1 components in magnetic field
- Resonance techniques
  - 2I+1 different orientations
- Quadrupole Moments: \( q = \frac{2}{5}Z(a^2-c^2) \), \( R^2 = \frac{1}{2}(a^2 + c^2) = (r_0A^{1/3})^2 \)
  - Data in barns, can solve for \( a \) and \( c \)
- Only nuclei with \( I > 1/2 \) have quadrupole moments
  - Non-spherical nuclei
  - Interactions of nuclear quadrupole moments with the electric fields produced by electrons in atoms and molecules give rise to abnormal hyperfine splittings in spectra
- Methods of measurement: optical spectroscopy, microwave spectroscopy, nuclear resonance absorption, and modified molecular-beam techniques
Spin and Parity

- System wave function sign change if sign of the space coordinates change
  - system has odd or even parity
- Parity is conserved
- even+odd=odd, even+even=even, odd+odd=odd
  - allowed transitions in atoms occur only between an atomic state of even and one of odd parity
- Parity is connected with the angular-momentum quantum number l
  - states with even l have even parity
  - states with odd l have odd parity

\[ P: \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix}. \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Charge</th>
<th>Rest Mass</th>
<th>Spin</th>
<th>Magnetic Moment</th>
<th>Statistics</th>
</tr>
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<tr>
<td>( e^- ), ( \beta^- )</td>
<td>Electron</td>
<td>-1</td>
<td>0.0005486</td>
<td>1/2</td>
<td>-1836</td>
<td>F</td>
</tr>
<tr>
<td>( e^+ ), ( \beta^+ )</td>
<td>Positron</td>
<td>+1</td>
<td>0.0005486</td>
<td>1/2</td>
<td>+1836</td>
<td>F</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Photon</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>B</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Neutrino</td>
<td>0</td>
<td>&lt;2 × 10^{-7}</td>
<td>1/2</td>
<td>&lt;0.3</td>
<td>F</td>
</tr>
<tr>
<td>( n )</td>
<td>Neutron</td>
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<td>1.0086650</td>
<td>1/2</td>
<td>-1.913</td>
<td>F</td>
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<td>1/2</td>
<td>±8.891</td>
<td>F</td>
</tr>
<tr>
<td>( \pi^\pm )</td>
<td>Pi-meson (pion)</td>
<td>±1</td>
<td>0.1498</td>
<td>0</td>
<td></td>
<td>B</td>
</tr>
<tr>
<td>( \pi^0 )</td>
<td>Pi-meson (pion)</td>
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<td>0.1449</td>
<td>0</td>
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<td>B</td>
</tr>
<tr>
<td>( p )</td>
<td>Proton</td>
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<td>1.0072765^a</td>
<td>1/2</td>
<td>+2.793</td>
<td>F</td>
</tr>
</tbody>
</table>

*Table 2-1 Properties of Some "Elementary" Particles*
Nuclear Properties Topic review

• Understand role of nuclear mass in reactions
  ▪ Use mass defect to determine energetics
  ▪ Binding energies, mass parabola, models

• Determine Q values

• How are nuclear shapes described and determined
  ▪ Potentials
  ▪ Nucleon distribution

• Quantum mechanical terms
  ▪ Used in description of nucleus
Nuclear Properties Study Questions

• What do binding energetics predict about abundance and energy release?
• Determine and compare the alpha decay Q values for 2 even and 2 odd Np isotopes. Compare to a similar set of Pu isotopes.
• What are some descriptions of nuclear shape?
• Construct a mass parabola for A=117 and A=50
• What is the density of nuclear material?
• Describe nuclear spin, parity, and magnetic moment
• Using the appropriate mass excess data calculate the following Q values for $^{212}$Bi. Show the reaction
  ▪ $\beta^-$ decay
  ▪ $\beta^+$ decay
  ▪ EC
  ▪ Alpha decay
  \[\text{Which decay modes are likely}\]
Outline

- Readings: Modern Nuclear Chemistry Chapter 3; Nuclear and Radiochemistry Chapters 4 and 5

- Radioactive decay kinetics
  - Basic decay equations
    - Concepts
    - Error evaluation
    - Specific activity
    - Lifetime and half life
  - Utilization of equations
    - Mixtures
    - Equilibrium
    - Branching
    - Bateman (decay chain)
    - Cross section
  - Natural radiation
  - Dating

- \[ N_t = N_0 e^{-\lambda t} \]
  - \( N \) = number of nuclei,
  - \( \lambda \) = decay constant,
  - \( t \) = time
  - Also works for \( A \) (activity) or \( C \) (counts)
    * \[ A_t = A_0 e^{-\lambda t}, \]
    * \[ C_t = C_0 e^{-\lambda t} \]

- \( A = \lambda N \)
- \( 1/\lambda = 1/(\ln2/t_{1/2}) = 1.443t_{1/2} = \tau \)

- Error
  - \( M \) is number of counts
Equations

• Parent Daughter decay

\[ N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1o} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + N_{2o} e^{-\lambda_2 t} \]

• Maximum daughter activity

\[ t = \frac{\ln\left(\frac{\lambda_2}{\lambda_1}\right)}{\lambda_2 - \lambda_1} \]

• Equilibrium

  ▪ Transient

  \[ \frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1} \]

  Secular

\[ N_2 \lambda_2 = N_1 \lambda_1 \]

\[ A_2 = A_1 \]
Many Decays

\[ \frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3 \]

- Can use the Bateman solution to calculate entire chain
- Bateman assumes only parent present at time 0

\[ N_n = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} + C_n e^{-\lambda_n t} \]

\[ C_1 = \frac{\lambda_1 \lambda_2 \ldots \lambda_{n-1}}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)\ldots(\lambda_n - \lambda_1)} N_{1o} \]

\[ C_2 = \frac{\lambda_1 \lambda_2 \ldots \lambda_{n-1}}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)\ldots(\lambda_n - \lambda_2)} N_{1o} \]

ERG Nuclide program

- Program being upgraded
- Version 2.0.0.2 available from lecture link
  - [http://radchem.nevada.edu/classes/rfss/Lecture%20Decay%20kinetics.html](http://radchem.nevada.edu/classes/rfss/Lecture%20Decay%20kinetics.html)
  - In web links
    - As ERG Nuclides 2.0.0.2 local download
- Examples (atoms and activity)
  - $^{226}_{\text{Ra}}$
  - $^{95}_{\text{Zr}}$
  - $^{152}_{\text{Pr}}$
Branching decay

- Branching Decay
  - partial decay constants must be considered
    → Isotope has only one half life
  - if decay chain branches and two branches are later rejoined, branches are treated as separate chains
    → production of common member beyond branch point is sum of numbers of atoms formed by the two paths

- Branching ratio is based on relative constants
  - $\frac{\lambda_i}{\lambda_t}$ is the % of the decay branch

- $^{64}\text{Cu}$ relative half lives
Equations for production reactions: Cross Sections

• Probability of a nuclear process is generally expressed in terms of a cross section $\sigma$
  ▪ dimensions of an area

• Originates from probability for reaction between nucleus and impinging particle is proportional to the cross-sectional target area presented by the nucleus
  ▪ Doesn’t hold for charged particles that have to overcome Coulomb barriers or for slow neutrons

• Total cross section for collision with fast particle is never greater than twice the geometrical cross-sectional area of the nucleus
  ▪ cross section $\sigma$ is close to 1 barn for this case

• $10^{-24}$ cm$^2$=1 barn
Production of radionuclides

- $\sigma =$ cross section
- $\phi =$ neutron flux
- $t =$ time of irradiation

$$N_1 = \frac{N_0 \sigma \phi}{\lambda_1} (1 - e^{-\lambda_1 t})$$

$$(1 - e^{-\lambda t})$$

*maximum level (saturation factor)*

- Activity of radioactive product at end bombardment is divided by saturation factor, formation rate is obtained

$$R = A / (1 - e^{-\lambda t})$$
Nuclei production: Long irradiation compared to half-life

- Find amount of $^{56}$Mn ($t_{1/2} = 2.578$ hr, $\lambda = 7.469 \times 10^{-5}$ s$^{-1}$) from irradiation of 1 g of Mn in a neutron flux of $1 \times 10^{13}$ n/cm$^2$/s for 1 hour
  - $^{55}$Mn($n,\gamma$)$^{56}$Mn: $^{55}$Mn + n $\rightarrow$ $\gamma$ + $^{56}$Mn $\sigma = 13.3 \times 10^{-24}$ cm$^2$
  - $N_0 = 1 \text{g}/54.93804 \text{g/mol} \times 6.02 \times 10^{23} \text{atom/mol}$
  - $N_0 = 1.096 \times 10^{22}$ atom
- $R = 1 \times 10^{13}$ n/cm$^2$/s $\times 13.3 \times 10^{-24}$ cm$^2$ $\times 1.096 \times 10^{22}$ atom
  - $R = 1.457 \times 10^{12}$ atoms/sec
- $5.247 \times 10^{15}$ atoms $^{56}$Mn in 1 hour (does not account for decay)

$$N_1 = \frac{1.096 \times 10^{22} (13.3 \times 10^{-24} \times 1 \times 10^{13})}{7.469 \times 10^{-5}} \left(1 - e^{-7.469 \times 10^{-5} \times 3600}\right) N_1 = \frac{N_0 \sigma \phi}{\lambda_1} \left(1 - e^{-\lambda_1 t}\right)$$

$$N_1 = \frac{1.458 \times 10^{12}}{7.469 \times 10^{-5}} \left(1 - 7.642 \times 10^{-1}\right)$$

$$N_1 = 1.952 \times 10^{16} (2.358 \times 10^{-1}) = 4.603 \times 10^{15} \text{atoms}$$
Formation rate from activity

- \( R = \frac{A}{1-e^{-\lambda t}} \)
- \( 4.603 \times 10^{15} \) atoms \(^{56}\text{Mn} \) (\( t_{1/2} = 2.578 \) hr, \( \lambda = 7.469 \times 10^{-5} \) s\(^{-1}\)) from 1 hour irradiation
- \( A = \lambda N = 4.603 \times 10^{15} \times 7.469 \times 10^{-5} = 3.436 \times 10^{11} \) Bq
- \( R = \frac{A}{1-e^{-\lambda t}} \)
- \( R = \frac{3.436 \times 10^{11}}{1-\exp(-7.469 \times 10^{-5} \times 3600)} \)
- \( R = 1.457 \times 10^{12} \) atom/sec

Compare

- Time for \(^{90}\text{Y}\) to have same activity as \(^{90}\text{Sr}\) parent
  \( R = \frac{A}{1-e^{-\lambda t}} \)
- Neutron flux needed so fission on \(^{241}\text{Pu}\) is equal to decay to \(^{241}\text{Am}\)
Dating

- Radioactive decay as clock
  - Based on $N_t = N_0 e^{-\lambda t}$
    \[ t = \frac{\ln \frac{N_t}{N_0}}{-\lambda} = \frac{\ln \frac{N_0}{N_t}}{\lambda} \]
  - $N_0$ and $N_t$ are the number of radionuclides present at times $t=0$ and $t=t$
    - $N_t$ from $A = \lambda N$
- $t$ the age of the object
  - Need to determine $N_0$
    - For decay of parent $P$ to daughter $D$ total number of nuclei is constant
    \[ D(t) + P(t) = P_0 \]
Dating

- \( P_t = P_0 e^{-\lambda t} \)

- Measuring ratio of daughter to parent atoms
  - No daughter atoms present at \( t=0 \)
  - All daughter due to parent decay
  - No daughter lost during time \( t \)

- A mineral has a \( ^{206}\text{Pb}/^{238}\text{U} = 0.4 \). What is the age of the mineral?

\[
t = \frac{1}{\ln 2} \ln(1 + 0.4) \\
\frac{4.5E9a}{1}
\]

\( \rightarrow 2.2E9 \) years
Dating

- $^{14}$C dating
  - Based on constant formation of $^{14}$C
    - No longer uptakes C upon organism death
  - 227 Bq $^{14}$C/kgC at equilibrium
  - What is the age of a wooden sample with 0.15 Bq/g C?

\[
t = \frac{1}{\lambda} \ln\left(\frac{C_{eq}}{C_{sample}}\right)
\]

\[
t = \frac{1}{\ln 2} \ln\left(\frac{0.227}{0.15}\right) = \frac{1}{5730\ years} 
\]

\[
3420\ years
\]
Dating

- Determine when Oklo reactor operated
  - Today 0.7% $^{235}\text{U}$
  - Reactor 3.5% $^{235}\text{U}$
  - Compare $^{235}\text{U}/^{238}\text{U}$ ($U_r$) ratios and use $N_t = N_0 e^{-\lambda t}$

\[
U_r(t) = U_r(o) \frac{e^{-\lambda_{235}t}}{e^{\lambda_{238}t}} = U_r(o) e^{(-\lambda_{235}t + \lambda_{238}t)}
\]

\[
\ln \frac{U_r(t)}{U_r(o)} = t(-\lambda_{235} + \lambda_{238})
\]

\[
t = \frac{\ln \frac{7.05E-3}{3.63E-2}}{(-9.85E-10 + 1.55E-10)} = 1.97E9 \text{ years}
\]
Decay Kinetics Topic review

- Utilize and understand the basic decay equations
- Relate half life to lifetime
- Understand relationship between count time and error
- Utilization of equations for mixtures, equilibrium and branching
- Use cross sections for calculation nuclear reactions and isotope production
- Utilize the dating equation for isotope pair
Decay kinetics Study Questions

• Compare and contrast nuclear decay kinetics and chemical kinetics.
• If M is the total number of counts, what is the standard deviation and relative error from the counts?
• Define Curie and Becquerel
• How can half-life be evaluated?
• What is the relationship between the decay constant, the half-life, and the average lifetime?
• For an isotope the initial count rate was 890 Bq. After 180 minutes the count rate was found to be 750 Bq. What is the half-life of the isotope?
• A 0.150 g sample of $^{248}$Cm has a alpha activity of 0.636 mCi. What is the half-life of $^{248}$Cm?
• What is the half life for each decay mode for the isotope $^{212}$Bi?
• How are cross sections used to determine isotope production rate?
• Determine the amount of $^{60}$Co produced from the exposure of 1 g of Co metal to a neutron flux of $10^{14}$ n/cm²/sec for 300 seconds.
• What are the basic assumptions in using radionuclides for dating?
• How much activity for an experiment?
Decay Kinetics Pop Quiz

• You have a source that is 0.3 Bq and the source is detected with 50 % efficiency. It is counted for 10 minutes. Which total counts shown below are not expected from these conditions?
  • 95, 81, 73, 104, 90, 97, 87

Useful projects

• Make excel sheets to calculate
  ▪ Mass or mole to activity
    → Calculate specific activity
  ▪ Concentration and volume to activity
    → Determine activity for counting
  ▪ Isotope production from irradiation
  ▪ Parent to progeny
    → Daughter and granddaughter
      * i.e., $^{239}\text{U}$ to $^{239}\text{Np}$ to $^{239}\text{Pu}$
Useful projects

• Make excel sheets to calculate
  ▪ Mass or mole to activity
    → Calculate specific activity
  ▪ Concentration and volume to activity
    → Determine activity for counting
  ▪ Isotope production from irradiation
  ▪ Parent to progeny
    → Daughter and granddaughter
      * i.e., $^{239}\text{U}$ to $^{239}\text{Np}$ to $^{239}\text{Pu}$
Lecture 4 Alpha Decay

- Readings
  - Nuclear and Radiochemistry: Chapter 3
  - Modern Nuclear Chemistry: Chapter 7

- Energetics of Alpha Decay
  - Geiger Nuttall based theory

- Theory of Alpha Decay

- Hindrance Factors
  - Different between theory and measurement

- Heavy Particle Radioactivity

- Proton Radioactivity

- Identified at positively charged particle Rutherford
  - Helium nucleus (\(^{4}\text{He}^{2+}\)) based on observed emission bands
  - Energetics
    - Alpha decay energies 4-9 MeV
    - Originally thought to be monoenergetic, fine structure discovered

- \(^{A}Z \rightarrow (A-4)(Z-2) + ^{4}\text{He} + Q_{\alpha}\)

Alpha decay observed for negative binding energies
**Energetics**

- Q values generally increase with A
  - variation due to shell effects can impact trend increase
  - Peaks at N=126 shell
- For isotopes decay energy generally decreases with increasing mass
- 82 neutron closed shell in rare earth region
  - increase in $Q_\alpha$
  - $\alpha$-decay for nuclei with N=84 as it decays to N=82 daughter
- short-lived $\alpha$-emitters near doubly magic $^{100}$Sn
  - $^{107}$Te, $^{108}$Te, $^{111}$Xe
- alpha emitters have been identified by proton dripline above A=100

\[
Q = T_\alpha + T_d
\]
\[
T_d = \frac{m_\alpha T_\alpha}{m_d}
\]
\[
Q = T_\alpha + \frac{m_\alpha T_\alpha}{m_d}
\]
\[
Q = T_\alpha \left( 1 + \frac{m_\alpha}{m_d} \right)
\]
\[
\frac{Q}{(1 + \frac{m_\alpha}{m_d})} = T_\alpha = Q\left( \frac{m_d}{m_\alpha + m_d} \right)
\]
Energetics

• Calculation of Q value from mass excess
  - \( ^{238}\text{U} \rightarrow ^{234}\text{Th} + \alpha + Q \)
  - Isotope \( \Delta \) (MeV)
    - \(^{238}\text{U} \) \( 47.3070 \)
    - \(^{234}\text{Th} \) \( 40.612 \)
    - \(^{4}\text{He} \) \( 2.4249 \)
  - \( Q_{\alpha} = 47.3070 - (40.612 + 2.4249) = 4.270 \) MeV
  - Q energy divided between \( \alpha \) particle and heavy recoiling daughter
    - kinetic energy of alpha particle will be slightly less than Q value

• Conservation of momentum in decay, daughter and alpha are equal \( \rho_d = \rho_{\alpha} \)
  - recoil momentum and \( \alpha \)-particle momentum are equal in magnitude and opposite in direction
  - \( p^2 = 2mT \) where \( m = \) mass and \( T = \) kinetic energy

• \(^{238}\text{U} \) alpha decay energy
  - \( T_\alpha = 4.720\left(\frac{234}{4 + 234}\right) = 4.198 \) MeV
  - \( T_\alpha = Q\left(\frac{m_d}{m_\alpha + m_d}\right) \)

• Is this the same as the measured alpha decay energy?
Energetics

- Kinetic energy of emitted particle is less than Coulomb barrier
  $\alpha$-particle and daughter nucleus
  - Equation specific of alpha
  - Particles touching
  - For $^{238}$U decay

\[
V_c = \frac{2Z_d e^2}{4\pi\varepsilon_0 R} = \frac{2Z_d}{1.2\left(A_d^{1/3} + 4^{1/3}\right)} 1.44\text{MeV fm}
\]

\[
V_c = \frac{2(90)}{1.2(234^{1/3} + 4^{1/3})}\text{fm} 1.44\text{MeV fm} \approx \frac{259\text{MeV fm}}{9.3 \text{ fm}} = 28\text{MeV}
\]

- Alpha decay energies are small compared to required energy for reverse reaction
- Alpha particle carries as much energy as possible from Q value,
- For even-even nuclei, alpha decay leads to ground state of daughter nucleus
  - as little angular momentum as possible
  - ground state spins of even-even parents, daughters and alpha particle are $l=0$
• Distance of closest approach for scattering of a 4.2 MeV alpha particle is ~62 fm
  ▪ Distance at which alpha particle stops moving towards daughter
  ▪ Repulsion from Coulomb barrier

• Alpha particle should not get near nucleus
  ▪ should be trapped behind a potential energy barrier

• Wave functions are only completely confined by infinitely high potential energy barriers
  ▪ With finite size barrier wave function has different behavior
  ▪ main component inside barrier
  ▪ finite piece outside barrier

• Tunneling
  ▪ trapped particle has component of wave function outside potential barrier
  ▪ Some probability to go through barrier
    → Related to decay probability
  ▪ Higher energy has higher tunneling probability
Alpha Decay Theory

- Closer particle energy to barrier maximum more likely particle will penetrate barrier

- More energetic alpha will encounter barrier more often
  
  \[ T = \frac{1}{2} \frac{mv^2}{Q_a} \]

  - Increase probability of barrier penetration due

- Geiger Nuttall law of alpha decay
  
  \[ \log t_{1/2} = A + \frac{B}{\sqrt{Q_a}} \]

  - Constants A and B have Z dependence.

- Simple relationship describes data on \( \alpha \)-decay
  
  - Over 20 orders of magnitude in decay constant or half-life
  
  - 1 MeV change in \( \alpha \)-decay energy results in a change of 10^5 in half-life
Expanded Alpha Half Life Calculation

• More accurate models of half life are possible
  ▪ Example from Hatsukawa, Nakahara and Hoffman
    \[
    \log_{10}(t_{1/2}) = A(Z)\left(\frac{A_d}{A_pQ_\alpha}\right)^{1/2}\arccos\sqrt{X - \sqrt{X(1 - X)}} - 20.446 + C(Z, N)
    \]
    \[C(Z, N) = 0 \quad \text{Outside of closed shells}
    \]
    \[C(Z, N) = [1.94 - 0.020(82 - Z) - 0.070(126 - N) \quad 78 \leq Z \leq 82; 100 \leq N \leq 126
    \]
    \[C(Z, N) = [1.42 - 0.105(Z - 82) - 0.067(126 - N) \quad 82 \leq Z \leq 90; 100 \leq N \leq 126
    \]
    \[X = 1.2249\left(A^{1/3} + 4^{1/3}\right)(\frac{Q_\alpha}{2Z_d e^2})
    \]

• Theoretical description of alpha emission based on calculating rate in terms of two factors
  ▪ rate at which an alpha particle appears at inside wall of nucleus
  ▪ probability that alpha particle tunnels through barrier
    • \(\lambda_\alpha = P \cdot f\)
    → \(f\) is frequency factor
    → \(P\) is transmission coefficient
Alpha Decay Theory

- Now have additional factor that describes probability of preformation of alpha particle inside parent nucleus prior to decay
- No clear way to calculate preformation probability
  - empirical estimates have been made
  - theoretical estimates of emission rates are higher than observed rates
    - uncertainties in theoretical estimates contribute to differences
  - preformation factor can be estimated for each measured case
- Evaluation of frequency for alpha particle to reach edge of a nucleus
  - estimated as velocity divided by distance across nucleus
    - twice radius, on order of fm
    - lower limit for velocity obtained from kinetic energy of emitted alpha particle
    - Use this to determine velocity of alpha particle in nucleus
    - particle is moving inside a potential energy well and its velocity should be larger and correspond to well depth plus external energy
  - On order of $10^{21} \text{s}^{-1}$

\[ f = \frac{v}{2R} \approx \frac{\sqrt{2(V_0 + Q) / \mu}}{2R} \]

\[ \mu = \frac{M_\alpha M_d}{M_\alpha + M_d} \]

Reduced mass
Alpha Decay Calculations

• Alpha particle barrier penetration from Gamow
  - \( T = e^{-2G} \)

• Determination of decay constant from potential information
  - Using square-well potential, integrating and substituting
    - Z daughter, z alpha

\[
\lambda = \frac{h}{2\mu R_1^2} \exp \left[ - \frac{4\pi}{h} (2\mu)^{1/2} \int_{R_4}^{R_2} (U(r) - T)^{1/2} \, dr \right]
\]

\[
\mu = \frac{M_\alpha M_d}{M_\alpha + M_d}
\]

\[
T = \frac{Zze^2}{R_2} = \frac{1}{2} \mu \nu^2
\]

\[
B = \frac{Zze^2}{R_1}
\]

\[
\lambda = \frac{h}{2\mu R_1^2} \exp \left[ - \frac{8\pi Zze^2}{h\nu} \arccos \left( \frac{T}{B} \right)^{1/2} - \left( \frac{T}{B} \right)^{1/2} \left( 1 - \frac{T}{B} \right)^{1/2} \right]
\]
Gamow calculations

\[ t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{fP} = \frac{\ln 2}{\sqrt{\frac{2(V_o + Q_\alpha)}{\mu}}} e^{-2G} \]

- From Gamow
  \[ \log t_{1/2} = A + \frac{B}{\sqrt{Q_a}} \]

- Calculated emission rate typically one order of magnitude larger than observed rate
  - observed half-lives are longer than predicted
  - Observation suggest a route to evaluate alpha particle pre-formation factor
• Even-even nuclei undergoing $l=0$ decay
  - average preformation factor is $\approx 10^{-2}$
  - neglects effects of angular momentum
    - Assumes $\alpha$-particle carries off no orbital angular momentum ($\ell = 0$)
  - If $\alpha$ decay takes place to or from excited state some angular momentum may be carried off by $\alpha$-particle
  - Results in change in decay constant when compared to calculated
Hindered $\alpha$-Decay

- Previous derivation only holds for even-even nuclei
  - odd-odd, even-odd, and odd-even nuclei have longer half-lives than predicted due to hindrance factors
- Assumes existence of pre-formed $\alpha$-particles
  - Ground-state transition from nucleus containing odd nucleon in highest filled state can take place only if that nucleon becomes part of $\alpha$-particle
    - therefore another nucleon pair is broken
    - less favorable situation than formation of an $\alpha$-particle from already existing pairs in an even-even nucleus
      - may give rise to observed hindrance
  - $\alpha$-particle is assembled from existing pairs in such a nucleus, product nucleus will be in an excited state
    - this may explain higher probability transitions to excited states
- Hindrance from difference between calculation and measured half-life
  - Hindrance factors between 1 and 3E4
  - Hindrance factors determine by
    - ratio of measured alpha decay half life over calculated alpha decay half life
    - ratio of calculated alpha decay constant over measured alpha decay constant

\[
\frac{t_{1/2, \text{measured}}}{t_{1/2, \text{calculated}}} = \frac{\lambda\alpha_{\text{calculated}}}{\lambda\alpha_{\text{measured}}} = \text{Hindrance factor}
\]
Hindrance Factors

- Transition of $^{241}\text{Am} (5/2^-)$ to $^{237}\text{Np}$
  - States of $^{237}\text{Np} (5/2^+)$ ground state and $(7/2^+)$ 1st excited state have hindrance factors of about 500 (red circle)
  - Main transition to 60 keV above ground state is 5/2-, almost unhindered
Hindrance Factors

- 5 classes of hindrance factors based on hindrance values
- Hindrance factors increase with increasing change in spin
  - Parity change also increases hindrance factor
- Between 1 and 4, transition is called a “favored”
  - Emitted alpha particle is assembled from two low lying pairs of nucleons in parent nucleus, leaving odd nucleon in its initial orbital
- Hindrance factor of 4-10 indicates a mixing or favorable overlap between initial and final nuclear states involved in transition
- Factors of 10-100 indicate that spin projections of initial and final states are parallel, but wave function overlap is not favorable
- Factors of 100-1000 indicate transitions with a change in parity but with projections of initial and final states being parallel
- Hindrance factors of >1000 indicate that transition involves a parity change and a spin flip
Heavy Particle Decay

- Possible to calculate Q values for emission of heavier nuclei
  - Is energetically possible for a large range of heavy nuclei to emit other light nuclei.
- Q-values for carbon ion emission by a large range of nuclei
  - Calculated with smooth liquid drop mass equation without shell corrections
- Decay to doubly magic $^{208}\text{Pb}$ from $^{220}\text{Ra}$ for $^{12}\text{C}$ emission
  - Actually found $^{14}\text{C}$ from $^{222,223}\text{Ra}$
  - Large neutron excess favors emission of neutron-rich light products
  - Emission probability is much smaller than alpha decay
- Simple barrier penetration estimate can be attributed to very small probability to preform $^{14}\text{C}$ residue inside heavy nucleus
Proton Decay

- For proton-rich nuclei, Q value for proton emission can be positive
  - Line where $Q_p$ is positive, proton drip line
  - Describes forces holding nuclei together
- Similar theory to alpha decay
  - no preformation factor for proton
  - proton energies, even for heavier nuclei, are low ($E_p \approx 1$ to 2 MeV)
- barriers are large (80 fm)
  - Long half life

- Examine proton drip line
Alpha Decay Topic Review

- Understand and utilize systematics and energetics involved in alpha decay
- Calculate Q values for alpha decay
  - Relate to alpha energy and fine structure
- Correlate Q value and half-life
- Models for alpha decay constant
  - Tunneling and potentials
- Hindered of alpha decay
- Understand proton and other charged particle emission
Homework Questions

• Calculate alpha decay Q value and Coulomb barrier potential for following, compare values
  ▪ $^{212}\text{Bi}$, $^{210}\text{Po}$, $^{238}\text{Pu}$, $^{239}\text{Pu}$, $^{240}\text{Am}$, $^{241}\text{Am}$
• What is basis for daughter recoil during alpha decay?
• What is relationship between $Q_a$ and alpha decay energy ($T_a$)
• What are some general trends observed in alpha decay?
• Compare calculated and experimental alpha decay half life for following isotopes
  ▪ $^{238}\text{Pu}$, $^{239}\text{Pu}$, $^{241}\text{Pu}$, $^{245}\text{Pu}$
  ▪ Determine hindrance values for odd A Pu isotopes above
• What are hindrance factor trends?
• How would one predict half-life of an alpha decay from experimental data?
Pop Quiz

- Calculate alpha decay energy for $^{252}$Cf and $^{254}$Cf from mass excess data below.
- Which is expected to have shorter alpha decay half-life and why?
- Calculate alpha decay half-life for $^{252}$Cf and $^{254}$Cf from data below (use % alpha decay)

Provide response in blog
Send answer by e-mail or next class meeting

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>E(level) (MeV)</th>
<th>Jπ</th>
<th>Δ(MeV)</th>
<th>$T_{1/2}$</th>
<th>Abundance</th>
<th>Decay Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{252}\text{Cf}$</td>
<td>0.0000</td>
<td>0+</td>
<td>76.0340</td>
<td>2.645 y 8</td>
<td></td>
<td>α : 96.91 %</td>
</tr>
<tr>
<td>$^{254}\text{Cf}$</td>
<td>0.0000</td>
<td>0+</td>
<td>81.3408</td>
<td>60.5 d 2</td>
<td></td>
<td>SF : 3.09 %</td>
</tr>
<tr>
<td>$^{248}\text{Cm}$</td>
<td>0.0000</td>
<td>0+</td>
<td>67.3922</td>
<td>3.48E+5 y 6</td>
<td></td>
<td>α : 99.69 %</td>
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<tr>
<td>$^{250}\text{Cm}$</td>
<td>0.0000</td>
<td>0+</td>
<td>72.9890</td>
<td>≈ 8.3E+3 y</td>
<td></td>
<td>α : 0.31 %</td>
</tr>
<tr>
<td>$^4\text{He}$</td>
<td>0.0000</td>
<td>0+</td>
<td>2.4249</td>
<td>STABLE</td>
<td>99.999863% 3</td>
<td>α : 91.61 %</td>
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<td>SF : 8.39 %</td>
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<td>SF ≈ 74.00 %</td>
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<td></td>
<td>α ≈ 18.00 %</td>
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<td></td>
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<td></td>
<td>β^- ≈ 8.00 %</td>
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