Outline

- Radioactive decay kinetics
  - Basic decay equations
  - Utilization of equations
    - Mixtures
    - Equilibrium
    - Branching
  - Natural radiation
  - Dating
Introduction

- Number of radioactive nuclei that decay in a sample decreases with time
  - Exponential decrease
  - Independent of P, T, mass action
    - Conditions associated with chemical kinetics
      - Electron capture and internal conversion can be affected by conditions
  - Specific for isotope
  - Irreversible

- Decay of given radionuclide is random
  - Statistical
    - Evaluate behavior of group
Basic Decay Equations

• Decay is 1\textsuperscript{st} order
  ➡️ Rate proportional to amount of parent isotope
    ➡️ Equal to the rate of isotope disintegration
    ➡️ Proportional to number of radioactive nuclei
    * rate of decay = decay constant \times \#$ radioactive nuclei
  ➡️ Decay constant is average decay probability per nucleus for a unit time
  ➡️ Represented by $\lambda$
Basic decay equations

- The radioactive process is a subatomic change within the atom
- The probability of disintegration of a particular atom of a radioactive element in a specific time interval is independent of its past history and present circumstances
- The probability of disintegration depends only on the length of the time interval.

Probability of decay: \( p = \lambda \Delta t \)

Probability of not decaying: \( 1 - p = 1 - \lambda \Delta t \)
Statistics of Radioactive Decay

1-p = 1 - $\lambda \Delta t$ = probability that atom will survive $\Delta t$

$(1 - \lambda \Delta t)^n =$ probability that atom will survive $n$ intervals of $\Delta t$

$n \Delta t = t$, therefore $(1 - \lambda \Delta t)^n = (1 - \lambda t/n)^n$

Since $\lim_{n \to \infty} (1 + x/n)^n = e^x$, $(1 - \lambda t/n)^n = e^{-\lambda t}$, the limiting value.

Considering $N_0$ atoms, the fraction remaining unchanged after time $t$ is $N/N_0 = e^{-\lambda t}$

\[ N = N_0 e^{-\lambda t} \]

where $\lambda$ is the decay constant
Radioactivity as Statistical Phenomenon

• **Binomial Distribution for Radioactive Disintegrations**
  - Probability $W(m)$ of obtaining $m$ disintegrations in time $t$ from $N_0$ original radioactive atoms
    
    $$W(m) = \frac{N_0!}{(N_0 - m)!m!} p^m (1 - p)^{N_0 - m}$$

  - Probability of atom not decaying in time $t$, $1-p$, is $(N/N_0)=e^{-\lambda t}$, where $N$ is number of atoms that survive in time interval $t$ and $N_0$ is initial number of atoms

• **Time Intervals between Disintegrations**
  - Probability of time interval having value between $t$ and $t+d$: 
    $$P(t)dt = N_0 \lambda e^{-N_0 \lambda t} dt$$
• Average Disintegration Rate

\[ W(r) = \frac{n!}{(n-r)!r!} p^r q^{n-r} \]

where 1-p=q

\[ np = \sum_{r=0}^{\infty} rW(r) = \bar{r} \]

for radioactive disintegration--if n=No and p=1-e^{-\lambda t}--average number M of atoms disintegrating in time t is M=No(1-e^{-\lambda t}); for small \lambda t, M=No\lambda t and disintegration R=M/t=No \lambda, which corresponds to -dN/dt=\lambda N

• Expected Standard Deviation

\[ \sigma = \sqrt{N_o (1-e^{-\lambda t})e^{-\lambda t}} = \sqrt{Me^{-\lambda t}} \]

Since in counting practice \lambda t is generally small, \sigma = \sqrt{M}

• M is number of counts

• Relative error = \sigma^{-1}
Measured Activity

- In practicality, activity (A) is used instead of the number of atoms (N).

\[ A = c \lambda t, \ m \]

where \( c \) is the detection coefficient

\[ A = A_0 e^{-\lambda t} \]

- Units

\( \text{Curie} \)

\( \Rightarrow 3.7\times10^{10} \text{ decay/s} \)

\* 1 g Ra

\( \text{Becquerel} \)

\( \Rightarrow 1 \text{ decay/s} \)
Half Life and decay constant

Half-life is time needed to decrease nuclides by 50%.

Relationship between $t_{1/2}$ and $\lambda$

\[
\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda t}
\]

\[
\ln(1/2) = -\lambda t_{1/2}
\]

\[
\ln 2 = \lambda t_{1/2}
\]

\[
t_{1/2} = \frac{\ln 2}{\lambda}
\]
Half lives

- Large variation in half-lives for different isotopes
  - Short half-lives can be measured
    - Evaluate activity over time
      * Observation on order of half-life
  - Long half-lives
    - Based on decay rate and sample
      * Need to know total amount of nuclide in sample
      * $A = \lambda n$
    - A is activity, n is number of nuclei
Exponential Decay

• **Average Life (τ)** for a radionuclide

  found from sum of times of existence of all atoms divided by initial number of nuclei

  \[
  \tau = -\frac{1}{N_o} \int_{t=0}^{t=\infty} t \cdot dN = \frac{1}{\lambda}
  \]

  \(1/\lambda = 1/(\ln2/t_{1/2}) = 1.443t_{1/2} = \tau\)

  Average life greater than half life by factor of 1/0.693

  during time 1/\(\lambda\) activity reduced to 1/e it’s initial value
Lifetime

- Total number of nuclei that decay over time
  - Dose
  - Atom at a time
- Couple with Heisenberg uncertainty principle
  - \( \Delta E \Delta t \geq \hbar/2\pi \)
    - \( \Delta t \) is \( \tau \)
    - with energy in eV
    - \( \Delta E \geq (4.133 \times 10^{-15} \text{ eV s}/2\pi)/\tau = \Gamma \)
    - \( \Gamma \) is decay width
      - * Resonance energy
        - \( t_{1/2} = 1 \text{ sec}, \tau = 1.44 \text{ s}, \Gamma = 4.56 \times 10^{-16} \text{ eV} \)
Width and energy

- Need very short half-lives for large widths
- Useful in Moessbauer spectroscopy
  - Absorption distribution is centered around $E_\gamma + \Delta E$
  - Emission centered $E_\gamma - \Delta E$.
- Overlapping part of the peaks can be changed by changing the temperature of the source and/or the absorber.
Equations

• \( N_t = N_0 e^{-\lambda t} \)
  
  \( N = \text{number of nuclei}, \ \lambda = \text{decay constant}, \ \text{t}=\text{time} \)

  ➞ Also works for \( A \) (activity) or \( C \) (counts)

  * \( A_t = A_0 e^{-\lambda t}, \ C_t = C_0 e^{-\lambda t} \)

• \( A = \lambda N \)

• \( 1/\lambda = 1/(\ln2/t_{1/2}) = 1.443t_{1/2} = \tau \)
Half-life calculation

Using $N_t = N_0 e^{-\lambda t}$

- For an isotope the initial count rate was 890 Bq. After 180 minutes the count rate was found to be 750 Bq

What is the half-life of the isotope

\[ 750 = 890 e^{-\lambda \times 180 \text{ min}} \]
\[ 750/890 = e^{-\lambda \times 180 \text{ min}} \]
\[ \ln(750/890) = -\lambda \times 180 \text{ min} \]
\[ -0.171/180 \text{ min} = -\lambda \]
\[ 9.5 \times 10^{-4} \text{ min}^{-1} = \lambda = \ln 2 / t_{1/2} \]
\[ t_{1/2} = \ln 2 / 9.5 \times 10^{-4} = 729.6 \text{ min} \]
Half-life calculation

\[ A = \lambda N \]

- A 0.150 g sample of \(^{248}\text{Cm}\) has a alpha activity of 0.636 mCi.

What is the half-life of \(^{248}\text{Cm}\)?

Find \( A \)

\[ \times 0.636 \times 10^{-3} \text{ Ci} (3.7 \times 10^{10} \text{ Bq/Ci}) = 2.35 \times 10^7 \text{ Bq} \]

Find \( N \)

\[ \times 0.150 \text{ g} \times 1 \text{ mole/248 g} \times 6.02 \times 10^{23} \text{ mole} = 3.64 \times 10^{20} \text{ atoms} \]

\[ \lambda = \frac{A}{N} = \frac{2.35 \times 10^7 \text{ Bq}}{3.64 \times 10^{20} \text{ atoms}} = 6.46 \times 10^{-14} \text{ s}^{-1} \]

\[ t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{6.46 \times 10^{-14} \text{ s}^{-1}} = 1.07 \times 10^{13} \text{ s} \]

\[ 1.07 \times 10^{13} \text{ s} = 1.79 \times 10^{11} \text{ min} = 2.99 \times 10^{9} \text{ h} = 1.24 \times 10^{8} \text{ d} \]

\[ = 3.4 \times 10^5 \text{ a} \]
Your gamma detector efficiency at 59 keV is 15.5 %. What is the expected gamma counts from 75 micromole of $^{241}$Am?

- Gamma branch is 35.9 % for $^{241}$Am
- $C = (0.155)(0.359)\lambda N$
- $t_{1/2} = 432.7\text{ a}\times (3.16E7\text{ s/a}) = 1.37E10\text{ s}$
- $\lambda = \frac{\ln 2}{1.37E10\text{ s}} = 5.08E-11\text{ s}^{-1}$
- $N = 75E-6\text{ moles } \times 6.02E23/\text{mole} = 4.52E19\text{ atoms}$
- $C = (0.155)(0.359)5.08E-11\text{ s}^{-1}\times 4.52E19 = 1.28E8\text{ Bq}$
Decay Scheme
Specific activity

• Activity of a given amount of radionuclide
  ✈️ Use $A = \lambda N$
  ➤ Use of carrier should be included

• SA of $^{226}\text{Ra}$
  ✈️ 1 g $^{226}\text{Ra}$, $t_{\frac{1}{2}} = 1599$ a
  ✈️ 1 g * 1 mole/226 g * 6.02E23 atoms/mole = 2.66E21 atom = N
  ✈️ $t_{\frac{1}{2}} = 1599$ a * 3.16E7 s/a = 5.05E10 s
    ➤ $\lambda = \ln2/ 5.05E10$ s = 1.37E-11 s\(^{-1}\)
  ✈️ $A = 1.37E-11$ s\(^{-1}\) * 2.66E21 = 3.7E10 Bq
Specific Activity

- 1 g $^{244}\text{Cm}$, $t_{1/2}=18.1$ a
  - $1 \text{ g} \times 1 \text{ mole/244 g} \times 6.02E23 \text{ atoms/mole} = 2.47E21 \text{ atom} = N$
  - $t_{1/2}=18.1 \text{ a} \times 3.16E7 \text{ s/a} = 5.72E8 \text{ s}$
    - $\lambda=\ln2/ 5.72E8 \text{ s} = 1.21E-9 \text{ s}^{-1}$
  - $A=1.21E-9 \text{ s}^{-1} \times 2.47E21 = 2.99E12 \text{ Bq}$

- Generalized equation for 1 g
  - $6.02E23/\text{Isotope mass} \times 2.19E-8/ t_{1/2} (\text{a})$
  - $1.32E16/(\text{Isotope mass} \times t_{1/2} (\text{a}))$
<table>
<thead>
<tr>
<th>Isotope</th>
<th>t 1/2 a</th>
<th>SA (Bq/g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 C</td>
<td>5715</td>
<td>1.65E+11</td>
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<tr>
<td>228 Th</td>
<td>1.91E+00</td>
<td>3.03E+13</td>
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<tr>
<td>232 Th</td>
<td>1.40E+10</td>
<td>4.06E+03</td>
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<tr>
<td>248 Cm</td>
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$^{14}\text{C}$

\[ y = \frac{m^2}{M_0} \]

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<thead>
<tr>
<th></th>
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<tr>
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<tr>
<td>$R$</td>
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</table>
Specific Activity

• Activity/mole
  \[ \text{N}=6.02 \times 10^{23} \]

• SA (Bq/mole) of \(^{129}\text{I}, t_{1/2}=1.57 \times 10^7 \text{ a} \]
  \[ t_{1/2}=1.57 \times 10^7 \text{ a} \times 3.16 \times 10^7 \text{ s/a} = 4.96 \times 10^{14} \text{ s} \]
  \[ \lambda=\ln(2)/4.96 \times 10^{14} \text{ s} = 1.397 \times 10^{-15} \text{ s}^{-1} \]
  \[ A=1.397 \times 10^{-15} \text{ s}^{-1} \times 6.02 \times 10^{23}=8.41 \times 10^8 \text{ Bq} \]

• Generalized equation
  \[ \text{SA (Bq/mole)}=1.32 \times 10^{16} / t_{1/2} \text{ (a)} \]
## Specific Activity

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<td>2.31E+12</td>
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<td>22 Na</td>
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<td>R</td>
<td>1</td>
<td>NA</td>
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</table>
SA with carrier

• 1E6 Bq of $^{152}$Eu is added to 1 mmole Eu.
  
  ✤ Specific activity of Eu (Bq/g)
  ✤ Need to find g Eu

  ➤ 1E-3 mole $\times$ 151.96 g/mole = 1.52E-1 g

  ➤ = 1E6 Bq/1.52E-1 g = 6.58E6 Bq/g

  * = 1E9 Bq/mole

• What is SA after 5 years

  ✤ $t_{1/2}$ = 13.54 a

  ➤ = $6.58E6 \times \exp((-\ln2/13.54)\times5)$ =

  * 5.09E6 Bq/g
Lifetime

- Atom at a time chemistry
- $^{261}$Rf lifetime
  - Find the lifetime for an atom of $^{261}$Rf
    - $t_{1/2} = 65$ s
    - $\tau = 1.443t_{1/2}$
    - $\tau = 93$ s
- Determines time for experiment
- Method for determining half-life
Mixtures of radionuclides

- **Composite decay**
  - Sum of all decay particles
  - Not distinguished by energy

- **Mixtures of Independently Decaying Activities**
  - If two radioactive species mixed together, observed total activity is sum of two separate activities:
    \[ A = A_1 + A_2 = c_1 \lambda_1 N_1 + c_2 \lambda_2 N_2 \]
  - Any complex decay curve may be analyzed into its components
  - Graphic analysis of data is possible
Can determine initial concentration and half-life of each radionuclide
$y = m_1 \cdot \exp(-m_2 \cdot M_0) + m_3 \cdot \exp(-m_4 \cdot t)$

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<td>$R$</td>
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</tbody>
</table>

$\lambda = 0.554$

$t_{1/2} = 1.25 \text{ hr}$

$l = 0.067$

$t_{1/2} = 10.4 \text{ hr}$
Parent – daughter decay

- Isotope can decay into radioactive isotope
  - Uranium decay series
  - Lower energy
  - Different properties
    - $A$
    - $Z$
    - Spin
    - Parity
- For a decay parent -> daughter
  - Rate of daughter formation dependent upon parent decay rate - daughter decay rate
Parent - daughter

• For the system 1 decays into 2
\[ \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \]

• Rearranging gives
\[ dN_2 + \lambda_2 N_2 \, dt = \lambda_1 N_1 \, dt \]

• Solve and substitute for \( N_1 \) using \( N_{1t} = N_{10} e^{-\lambda t} \)
\[ dN_2 + \lambda_2 N_2 \, dt = \lambda_1 N_{10} e^{-\lambda_1 t} \, dt \]

\( \blacklozenge \) Linear 1\textsuperscript{st} order differential equation

\( \Rightarrow \) Solve by integrating factors

• Multiply by \( e^{\lambda_2 t} \)
\[ e^{\lambda_2 t} \, dN_2 + \lambda_2 N_2 e^{\lambda_2 t} \, dt = \lambda_1 N_{10} e^{(\lambda_2 - \lambda_1) t} \, dt \]
\[ d(N_2 e^{\lambda_2 t}) = \lambda_1 N_{10} e^{(\lambda_2 - \lambda_1) t} \, dt \]
Parent-daughter

- **Integrate from** $t_0$ to $t$

\[ \int_{t_0}^{t} N_2 e^{\lambda_2 t} \, dt = \int_{t_0}^{t} \frac{\lambda_1 N_{1o} e^{(\lambda_2 - \lambda_1) t}}{\lambda_2 - \lambda_1} \]

\[ N_2 e^{\lambda_2 t} - N_{2o} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1o} (e^{(\lambda_2 - \lambda_1) t} - 1) \]

- **Multiply by** $e^{-\lambda_2 t}$ **and solve for** $N_2$

\[ N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1o} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + N_{2o} e^{-\lambda_2 t} \]

Growth of daughter from parent

Decay of initial daughter
Parent daughter

• Can solve equation for activity from $A = \lambda N$

$$A_2 = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} N_{1o} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + A_{2o} e^{-\lambda_2 t}$$

• Find maximum daughter activity based on $dN/dt=0$

$$\lambda_1 e^{-\lambda_1 t} = \lambda_2 e^{-\lambda_2 t}$$

• Solve for $t$

$$t = \frac{\ln(\frac{\lambda_2}{\lambda_1})}{(\lambda_2 - \lambda_1)}$$

• For $^{99m}\text{Tc}$ ($t_{1/2}=6.01$ h) from $^{99}\text{Mo}$ (2.75 d)

$$\lambda_{\text{Tc}} = 2.8 \text{ d}^{-1}, \quad \lambda_{\text{Mo}} = 0.25 \text{ d}^{-1}$$

$$\Rightarrow 0.94 \text{ d}$$
Half life relationships

• No daughter decay
  ✎ No activity of daughter
  ✎ Number of daughter atoms due to parent decay
  \[ N_2 = N_{1o} (1 - e^{-\lambda_1 t}) \]

• No Equilibrium
  ✎ if parent is shorter-lived than daughter
  \((\lambda_1 > \lambda_2)\), no equilibrium attained at any time
  ✎ daughter reaches maximum activity when
  \[ \lambda_1 N_1 = \lambda_2 N_2 \]
  ➤ All parents decay, then decay is based on daughter
Half life relationships

• Transient equilibrium
  - Parent half life greater than 10 x daughter half life
    \[ \lambda_1 < \lambda_2 \]
  - Parent daughter ratio becomes constant over time
    - As t goes toward infinity
      \[ e^{-\lambda_2 t} \ll e^{-\lambda_1 t}; N_{2o} e^{-\lambda_2 t} \rightarrow 0 \]

\[
\begin{align*}
N_2 & \approx \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{1o} e^{-\lambda_1 t} \\
N_1 & = N_{1o} e^{-\lambda_1 t}
\end{align*}
\]

\[
\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1}
\]
Transient equilibrium: (a) total activity of an initially pure parent fraction; (b) activity parent \( t_{1/2} = 8.0 \text{ h} \); (c) decay of freshly isolated daughter fraction \( t_{1/2} = 0.80 \text{ h} \); (d) activity growing in freshly purified parent fraction; (e) total daughter activity in lus-daughter fractions.
Half life relationship

- Secular equilibrium
  - Parent much longer half-life than daughter
    - $1 \times 10^{4}$ times greater
    - ($\lambda_1 << \lambda_2$)
  - Parent activity does not measurably decrease in many daughter half-lives

\[
\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1} \quad \frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2}
\]

\[
N_2 \lambda_2 = N_1 \lambda_1 \\
A_2 = A_1
\]
Fig. 5.3 Secular equilibrium: (a) total activity of an initially pure parent fraction; (b) activity to parent ($t_{1/2} = \infty$); this is also the total daughter activity in parent-plus-daughter fraction; decay of freshly isolated daughter fraction ($t_{1/2} = 0.80$ h); (d) daughter activity growing in purified parent fraction.
Many Decays

\[
\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3
\]

- Bateman solution
- Only parent present at time 0

\[
N_n = C_1 e^{-\lambda_1 t} + C_2 e^{-\lambda_2 t} + C_{n} e^{-\lambda_n t}
\]

\[
C_1 = \frac{\lambda_1 \lambda_2 \ldots \lambda_{(n-1)}}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1) \ldots (\lambda_n - \lambda_1)} N_{10}
\]

\[
C_2 = \frac{\lambda_1 \lambda_2 \ldots \lambda_{(n-1)}}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2) \ldots (\lambda_n - \lambda_2)} N_{10}
\]
Branching decay

- Branching Decay
  - partial decay constants must be considered
    - $A$ has only one half life
      $$\lambda = \sum_{i=1}^{N} \lambda_i \cdot \frac{1}{t_{1/2}} = \sum_{i=1}^{N} \frac{1}{t_{1/2}}$$
  - if decay chain branches and two branches are later rejoined, the two branches are treated as separate chains
    - production of common member beyond branch point is sum of numbers of atoms formed by the two paths
- Branching ratio is based on relative constants
  - $\frac{\lambda_i}{\lambda_t}$
Branching Decay

• For a branching decay of alpha and beta

- \[ A_t = A_\alpha + A_\beta \]
  - \[ A = \lambda N, \text{ so} \]
  - \[ \lambda_t N = \lambda_\alpha N + \lambda_\beta N; \lambda_t = \lambda_\alpha + \lambda_\beta \]

- \[ 1 = A_\alpha / A_t + A_\beta / A_t; 1 = \lambda_\alpha / \lambda_t + \lambda_\beta / \lambda_t \]

• Consider \(^{212}\)Bi, what is the half life for each decay mode?

- Alpha branch 36 %, beta branch 64 %
- \[ t_{1/2} = 60.55 \text{ min} \]
  - \[ \lambda_t = 0.0114; 0.36 = \lambda_\alpha / \lambda_t; 0.36 = \lambda_\alpha / 0.0114; \lambda_\alpha = 0.0041 \]
  - \[ t_{1/2} \text{ alpha} = 169 \text{ min} \]
- \[ \lambda_t = \lambda_\alpha + \lambda_\beta; 0.0114 = 0.0041 + \lambda_\beta; 0.0073 = \lambda_\beta \]
  - \[ t_{1/2} \text{ beta} = 95.0 \text{ min} \]
Abb. 1.1: Radiotoxizität hochradioaktiver Abfälle von Kernbrennstoffen aus Leichtwasserreaktoren nach einem Abbrand von 33 GWd/t ohne Wiederaufarbeitung.
Cross Sections

The probability of a nuclear process is generally expressed in terms of a cross section $\sigma$ that has the dimensions of an area.

- Originates from simple picture that probability for reaction between nucleus and impinging particle is proportional to the cross-sectional target area presented by the nucleus
  - doesn’t hold for charged particles that have to overcome Coulomb barriers or for slow neutrons

- Total cross section for collision with fast particle is never greater than twice the geometrical cross-sectional area of the nucleus
  - $10^{-24}$ cm$^2$=1 barn
For a beam of particles striking a thin target--one in which the beam is attenuated only infinitesimally--the cross section for a particular process is defined:

$$R_i = Inx\sigma_i$$

When a sample is embedded in a uniform flux of particles incident on it from all direction, such as in a nuclear reactor, the cross section is defined:

$$R_i = \phi N\sigma_i$$

$R_i =$ # of processes of type under consideration occurring in the target per unit time

$I =$ # of incident particles per unit time

$\phi =$ flux of particles/cm²/sec

$N =$ number of nuclei contained in sample

$n =$ # of nuclei/cm³

$x =$ target thickness (cm)
Production of radionuclides

- \( N_1 = N_0 \sigma \phi \)
  - \( \sigma \) = cross section
  - \( \phi \) = neutron flux

- To fully consider produced nuclei

\[
N_1 = \frac{N_0 \sigma \phi}{\lambda_1} (1 - \exp(-\lambda_1 t))
\]

\( t \) = time of irradiation

\( (1 - \exp(-\lambda_1 t)) \) gives maximum level percent

<table>
<thead>
<tr>
<th>half life</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>87.5</td>
</tr>
<tr>
<td>4</td>
<td>93.75</td>
</tr>
<tr>
<td>5</td>
<td>96.875</td>
</tr>
</tbody>
</table>
Natural Radionuclides

- 70 naturally occurring radioactive isotopes
  - Mainly decay from actinides
  - Tritium
  - $^{14}$C
  - $^{40}$K

- 70 kg “reference man,”
  - 4400 Bq of $^{40}$K
  - 3600 Bq of $^{14}$C

- US diet
  - 1 pCi/day of $^{238}$U, $^{226}$Ra, and $^{210}$Po

- air
  - ~0.15 pCi/L of $^{222}$Rn

- earth’s crust
  - ~10 ppm and ~4 ppm of the radioelements Th and U.

- interior heat budget of the planet Earth is dominated by the contributions from the radioactive decay of U, Th, and K
Environmental radionuclides

• primordial nuclides that have survived since the time the elements were formed
  ✧ $t_{1/2}>1\times10^9$ a
  ✧ Decay products of these long lived nuclides
    ➤ $^{40}$K., $^{87}$Rn, $^{238}$U, $^{235}$U, $^{232}$Th

• cosmogenic are shorter lived nuclides formed continuously by the interaction of cosmic rays with matter
  ✧ $^3$H., $^{14}$C, $^7$Be
    ➤ $^{14}$N(n, $^1$H)$^{14}$C (slow n)
    ➤ $^{14}$N(n, $^3$H)$^{12}$C (fast n)

• anthropogenic are nuclides introduced into the environment by the activities of man
  ✧ Actinides and fission products
  ✧ $^{14}$C and $^3$H
Dating

- Radioactive decay as clock
  - Based on $N_t = N_0 e^{-\lambda t}$
  - Solve for $t$
    \[
    t = \frac{\ln \frac{N_t}{N_0}}{-\lambda} = \frac{\ln \frac{N_0}{N_t}}{\lambda}
    \]

- $N_0$ and $N_t$ are the number of radionuclides present at times $t=0$ and $t=t$
  - $N_t$ from $A = \lambda N$

- $t$ the age of the object
  - Need to determine $N_0$

- For decay of parent $P$ to daughter $D$ total number of nuclei is constant
  \[
  D(t) + P(t) = P_0
  \]
Dating

- $P_t = P_0 e^{-\lambda t}$
- $t = \frac{1}{\lambda} \ln(1 + \frac{D_t}{P_t})$

- Measuring ratio of daughter to parent atoms
  - no daughter atoms present at $t=0$
  - that they are all due to the parent decay
  - none have been lost during time $t$

- A mineral has a $^{206}\text{Pb}/^{238}\text{U} = 0.4$. What is the age of the mineral?
  - $t = (1/(\ln2/4.5\times10^9))\ln(1+0.4)$
  - $\approx 2.2 \times 10^9$ years
Dating

- $^{14}$C dating
  - Based on constant formation of $^{14}$C
  - No longer uptakes C upon organism death
- 227 Bq $^{14}$C/kgC at equilibrium
- What is the age of a wooden sample with 0.15 Bq/g C?
  - $t = \frac{1}{\lambda} \ln\left(\frac{^{14}C_{eq}}{^{14}C_{sample}}\right)$
  - $t = \left(\frac{1}{(\ln2/5730 \text{ a})}\right) \times \ln(0.227/0.15) = 3420 \text{ a}$
Dating

- Determine when Oklo reactor operated
  - Today 0.7 % $^{235}$U
  - Reactor 3.5 % $^{235}$U
  - Compare $^{235}$U/$^{238}$U ($U_r$) ratios and use $N_t = N_0 e^{-\lambda t}$

$$U_r(t) = U_r(o) \frac{e^{-\lambda_{235}t}}{e^{-\lambda_{238}t}} = U_r(o)e^{(-\lambda_{235}t + \lambda_{238}t)}$$

$$\ln \frac{U_r(t)}{U_r(o)} = t(-\lambda_{235} + \lambda_{238})$$

$$t = \frac{\ln \frac{7.25E-3}{3.63E-2}}{(-9.85E-10 + 1.55E-10)} 1.94E9 \text{ years}$$
Questions

• Make excel sheets to calculate
  ➤ Mass or mole to activity
  ➤ Calculate specific activity
  ➤ Concentration and volume to activity
  ➤ Determine activity for counting
  ➤ Parent to progeny
    ➤ Daughter and granddaughter
      * i.e., $^{239}$U to $^{239}$Np to $^{239}$Pu