CHEM 312: Lecture 9 Part 1 Nuclear Reactions

- Readings: Modern Nuclear Chemistry, Chapter 10; Nuclear and Radiochemistry, Chapter 4
- Notation
- Energetics of Nuclear Reactions
- Reaction Types and Mechanisms
  - Barriers
  - Scattering
- Nuclear Reaction Cross Sections
- Reaction Observables
- Scattering
- Direct Reactions
- Compound Nuclear Reactions
- Photonuclear Reactions
- Nucleosynthesis
Nuclear Reactions

- Nucleus reactions with a range of particles
  - nucleus, subatomic particle, or photon to produce other nuclei
  - Short time frame (picosecond)
- First nuclear reaction from Rutherford
  - What reaction was this?
- Number of terms conserved during nuclear reactions
  - Number of nucleons
    - except in reactions involving creation or annihilation of antinucleons
  - charge
  - Energy
  - momentum
  - angular momentum
  - parity
- Q is the energy of the reaction
  - positive Q corresponds to energy release
  - negative Q to energy absorption
- Q terms given per nucleus transformed

\[ ^{14}_7N + ^4_2He \rightarrow ^{17}_8O + ^1_1H + Q \]
\[ ^{14}_7N(\alpha, p)^{17}_8O \]
Energetics

• Energetically many orders of magnitude greater than chemical reactions

• $^{14}\text{N}(\alpha,p)^{17}\text{O}$ $Q=-1.193$ MeV
  - Convert energy to per molar basis
    $\Rightarrow 1 \text{ MeV} = 1.60E-13 \text{ J}$

$$\frac{1.193 \text{MeV}}{\text{atom}} \times \frac{6.02E23 \text{ atoms}}{\text{mole}} \times \frac{1.6E-13 \text{ J}}{\text{MeV}} = 1.15E11 \frac{\text{J}}{\text{mole}}$$

• Reaction energies so large that mass change is observable
  - Chemical reactions in kJ/mole
Energetics

- Reaction Q values
  - Not necessarily equal to kinetic energy of bombarding particles for the reaction to occur
    - Need more energy than Q value for reaction to occur
      * Reaction products will have kinetic energy that needs to come from reaction

- Conservation of momentum
  - Some particles’ kinetic energy must be retained by products as kinetic energy

- Amount retained as kinetic energy of products
  - Based on projectile mass
  - Retained kinetic energy becomes smaller with increasing target mass

  → Equation for kinetic energy (T):

  $T = \frac{A_{\text{Projectile}}}{A_{\text{Projectile}} + A_{\text{Target}}} \times Q$

- What does this mean about reaction
  - Heavier target or heavier projectile?
    - $^{248}\text{Cm} + ^{18}\text{O} \rightarrow ^{266}\text{Rf}$

\[
T = \frac{248}{248 + 18} Q = 0.932Q \quad ^{248}\text{Cm Projectile}
\]
\[
T = \frac{18}{248 + 18} Q = 0.068Q \quad ^{18}\text{O Projectile}
\]
Energetics: Reaction Barrier

- Need to consider laboratory and center of mass frame
- **Laboratory frame**
  - Conservation of momentum considers angle of particles
  \[ Q = T_x (1 + \frac{m_x}{m_R}) - T_p (1 + \frac{m_p}{m_R}) - \frac{2}{m_R} \sqrt{(m_p T_p m_x T_x) \cos \theta} \]
- **Q value** can be found if \( T_x \) and \( \theta \) are measured and particles known
  - \( T_p \) from experiment
- **Center of mass**
  - Total particle angular momentum is zero
  \[ T_{cm} = \frac{(m_p + m_T) v_{cm}^2}{2} \quad v_{cm} = \frac{v_p m_p}{(m_p + m_T)} \]
- Kinetic energy carried by projectile (\( T_{lab} \)) is not fully available for reaction
  - \( T_{lab} - T_{cm} = T_0 \)
  - \( T_0 \) is energy to be dissipated in reaction
- For reaction to occur \( Q + T_0 \) must be achieved
  - Basis for threshold reaction
  - \( Q + T_0 \geq 0 \)
Reaction Barrier

- **Threshold energy (minimum energy for reaction)**

\[ Q + T_{\text{lab}} - T_{\text{CM}} \geq 0; \quad T_{\text{cm}} = T_{\text{lab}} \left( \frac{m_p}{m_p + m_T} \right) \]

Solve of laboratory \( T \)

\[ T_{\text{lab}} \geq \frac{-Q}{(1 - \left( \frac{m_p}{m_p + m_T} \right))} = \frac{-Q}{\left( \frac{m_p + m_T}{m_p + m_T} - \left( \frac{m_p}{m_p + m_T} \right) \right)} = \frac{-Q}{m_T} \]

\[ T \geq -Q \frac{A_{\text{Projectile}} + A_{\text{Target}}}{A_{\text{Target}}} \text{ MeV} \]

- **Fraction of bombarding particle’s kinetic energy retained as kinetic energy of products becomes smaller with increasing mass of target**
  - Heavier target or heavier projectile?
  - \( ^{248}\text{Cm} + ^{18}\text{O} \rightarrow ^{266}\text{Rf} \)
Consider the $^{14}\text{N}({\alpha,p})^{17}\text{O}$ reaction

- Find threshold energy
  \[ Q = 2.425 + 2.863 - 7.289 - (-0.809) = -1.19 \text{ MeV} \]
  \[ T \geq -Q = 1.53 \text{ MeV} \]

- Reaction barrier also induced by Coulomb interaction
  - Need to have enough energy to react and overcome Coulomb barrier
    \[ V_c = \frac{Z_1Z_2e^2}{R_1 + R_2} \quad R = r_0A^{1/3} \]
  - Equation can vary due to $r_0$
  - $V_c$ can be above threshold energy
    \[ V_c = 0.96 \frac{Z_1Z_2}{A_1^{1/3} + A_2^{1/3}} \text{ MeV} \]
    \[ V_c = 0.96 \times \frac{2}{4^{1/3} + 14^{1/3}} \text{ MeV} = 3.36 \text{ MeV} \]

- Center of mass, need to bring to laboratory frame
  - Consider kinetic energy carried by projectile
  - $3.36 \times \left(\frac{(14+4)}{14}\right) = 4.32$ MeV alpha needed for reaction
Cross Section Values and Limits

• Reaction cross section of $\pi R^2$ is approximated at high energies
  - Wave nature of incident particle causes upper limit of reaction cross section to include de Broglie wavelength
    → So cross section can be larger than area due to incoming particle wavelength
    → Expressed as an increase in $R$, quantum in nature
      
      $$\sigma_r = \pi (R + \lambda)^2$$

• Collision between neutron and target nucleus characterized by distance of closest approach
  - $B$ is impact parameter

Fig. 4-1 Collision with impact parameter $b$ between a neutron and target nucleus with interaction radius $R$. 
Cross sections

- Angular momentum of system is normal to the relative momentum $p$
  \[ L = pb = \frac{\hbar b}{\hat{\kappa}} = l\hbar \quad b = l\hat{\kappa} \]

- $b$ any value between 0 and $R$
  \[ l\hat{\kappa} < b < (l + 1)\hat{\kappa} \]

- $l = 0, 1, 2, \ldots b$ angular momentum
  - $l\hbar$

- Sum all $l$ from 0 to $l_{\text{max}}$
- Cross section based on summation of $l$ cross sections
- For this reason nuclear reaction cross sections can be several orders of magnitude larger than the nuclear geometrical cross section
  - Manifest by slow-neutron reactions

\[ \sigma_r = \pi (R + \hat{\kappa})^2 \]
Cross section

\[ \sigma_l = \pi \lambda^2 [(l+1)^2 - l^2] = \pi \lambda^2 (2l + 1) \]

- Quantum-mechanical treatment \( T_\ell \) is the transmission coefficient for reaction of a neutron with angular momentum \( \ell \)
  - Represents fraction of incident particles with angular momentum \( \ell \) that penetrate within range of nuclear forces
    - Provides summing term to increase cross section
    - Reason why cross section can be larger than physical size of nucleus

\[ \sigma_1 \] is partial cross section of given angular momentum \( l \)

- General trends for neutron and charged particles
  - Charged particle cross section minimal at low energy
  - Neutron capture cross section maximum at low energy

Fig. 9-2. The incident beam is perpendicular to the plane of the figure. The particles with a particular \( l \) are considered to strike within the designated ring.
Measuring Cross Section: Excitation Functions

- Variation of reaction cross section with incident energy
- Shape can be determined by exposing several target foils in same beam with energy-degrading
  - Simultaneous measurement of multiple particle energies
- Provide information about probabilities for emission of various kinds and combination of particles in nuclear reactions
  - Formation of given product implies what particles were ejected from target nuclide
- Range of cross sections can be evaluated
  - Detection limit of product can influence cross section limit measurement

Fig. 2. The $^{249}$Bk($^{48}$Ca, xn) excitation functions calculated in the framework of the model of Zagrebaev.
Barriers for Charged Particles

- Coulomb repulsion between charged bombarding particles and nucleus
  - Repulsion increases with decreasing distance of separation until charged particle comes within range of nuclear forces
  - Probability of tunneling through barrier drops rapidly as energy of particle decreases
  - Coulomb barriers affect charged particles both entering and leaving the nucleus
    - Charged particles emitted from nuclei experience Coulomb repulsion during emission
    - greater than 1 MeV
    - seen with position emission
- Related to change in cross section with energy for charged particle reactions
  - Maximum cross section dependent upon energy
CHEM 312: Lecture 9 Part 1 Nuclear Reactions

- Readings: Modern Nuclear Chemistry, Chapter 10; Nuclear and Radiochemistry, Chapter 4
- Notation
- Energetics of Nuclear Reactions
- Reaction Types and Mechanisms
  - Barriers
  - Scattering
- Nuclear Reaction Cross Sections
- Reaction Observables
- Scattering
- Direct Reactions
- Compound Nuclear Reactions
- Photonuclear Reactions
- Nucleosynthesis
CHEM 312: Lecture 9 Part 2 Nuclear Reactions

• Readings: Modern Nuclear Chemistry, Chapter 10; Nuclear and Radiochemistry, Chapter 4
• Notation
• Energetics of Nuclear Reactions
• Reaction Types and Mechanisms
  ▪ Barriers
  ▪ Scattering
• Nuclear Reaction Cross Sections
• Reaction Observables
  • Scattering
  • Direct Reactions
  • Compound Nuclear Reactions
  • Photonuclear Reactions
  • Nucleosynthesis
Reactions: Elastic Scattering

- Elastic scattering
  - kinetic energy conserved
  - Particles do not change
- Simplest consequence of a nuclear collision
  - Not a “reaction”
    → no exchange of nucleons or creation of particles
- Particles do not change their identity during the process and the sum of their kinetic energies remains constant
- Elastic scattering will also have a contribution from nuclear forces
Low-Energy Reactions with Light Projectiles

• Slow-Neutron Reactions
  ▪ Purest example of compound-nucleus behavior
    → $1/v$ law governs most neutron cross sections in region of thermal energies
  ▪ neutrons available only from nuclear reactions
    → Range of energy can be obtained

• Reaction Cross Sections
  ▪ Coulomb barrier prevents study of nuclear reactions with charged particles below 1 MeV
    → resonances no longer observable
    → with increasing energy, increasing variety of reactions possible
Low-Energy Reactions

• Deuteron Reactions
  ▪ Prevalence of one nucleon stripping
    → large size and loose binding of deuteron
    → Only proton and neutron in deuteron nucleus
      * Proton charge carries both nucleons
  ▪ Neutron comes within range of nuclear forces while proton is still outside most of Coulomb barrier
    → Inherent in large neutron-proton distance in deuteron
    → weakly bound deuteron can be broken up
      * proton outside barrier

• Competition among Reactions
  ▪ depends on relative probabilities for emission of various particles from compound nucleus
    → determined by number of factors
      * energy available
      * Coulomb barrier
      * density of final states in product nucleus
High Energy Reactions

- **Spallation Products**
  - Products in immediate neighborhood of target element found in highest yields
    - within 10 to 20 mass numbers
  - Yields tend to form in two regions
  - β stability for medium-weight products
  - Neutron-deficient side of stability with increasing Z of products
  - Used to produce beam of neutrons at spallation neutron source
    - Heavy Z will produce 20-30 neutrons
    - Basis of Spallation neutron source
      (http://neutrons.ornl.gov/facilities/SNS/)

- **High-Energy Fission**
  - Single broad peak in mass-yield curve instead of double hump seen in thermal-neutron fission
  - Many neutron-deficient nuclides
    - Especially among heavy products
    - Originate from processes involving higher deposition energies
    - Lower kinetic energies
    - Do not appear to have partners of comparable mass
    - Arise from spallation-like or fragmentation reactions
High-Energy Reactions

- Mass-Yield Curves
  - at low energies, compound-nucleus picture dominates
    - as energy increases importance of direct reactions and preequilibrium (pre-compound nucleus) emission increase
    - above 100 MeV, nuclear reactions proceed nearly completely by direct interactions
  - products down to mass number 150 are spallation products
  - those between mass numbers 60 and 140 are fission products

- Cascade-Evaporation Model
  - Above 100 MeV reactions
  - energy of the incident proton larger than interaction energy between the nucleons in the nucleus
  - Wavelength less than average distance between nucleons
    - proton will collide with one nucleon at a time within the nucleus
      * high-energy proton makes only a few collisions in nucleus
      * Produces nucleons with high energy
Heavy-Ion Reactions

- Range of heavy ion reactions
  - elastic and inelastic scattering
  - compound-nucleus formation,
  - direct interactions
  - deeply inelastic reaction

- Reactions influence by parameter
  - impact parameter of collision
  - kinetic energy of projectile
  - masses of target
  - projectile nuclei

- Elastic and Inelastic Scattering, Coulomb Excitation
  - elastic-scattering measurements used to obtain information on interaction radii
  - \( R = r_o \left( A_1^{1/3} + A_2^{1/3} \right) \) between mass numbers \( A_1 \) and \( A_2 \)
Heavy Ion Reactions

• **Inelastic scattering**
  - scattering in which some of projectile’s kinetic energy transformed into excitation of target nucleus
    - greatest importance at large impact parameters
  - heavy ions valuable
    - can excite high-spin states in target nuclei because of large angular momenta

• **Can experience Coulomb excitation**
  - high charges
  - below Coulomb barrier heights and excite nuclei by purely electromagnetic interactions

• **Transfer Reactions**
  - stripping and pickup reactions prevalent with heavy ions
    - take place at impact parameters just below those at which interactions are purely Coulombic
  - angular distributions show oscillatory, diffraction-like pattern when transfer reaction to single, well-defined state observed
Heavy Ion Reactions: Deep Inelastic Reactions

• Relatively large amounts of nuclear matter transferred between target and projectile
  ▪ Show strongly forward-peaked angular distributions
  ▪ “Grazing contact mechanism”

• Products with masses in vicinity of projectile mass appear at angles other than classical grazing angle
  ▪ Relatively small kinetic energies

• Total kinetic energies of products strongly correlated with amount of mass transfer
  ▪ Increasing mass difference of product and projectile lowers kinetic energy

• Product will dissociate into two fragments
  ▪ Appreciable fraction of incident kinetic energy dissipated and goes into internal excitation
Compound-Nucleus Reactions

• Compound-nucleus formation can only take place over a restricted range of small impact parameters
  ▪ can define critical angular momentum above which complete fusion cannot occur
  ▪ $\sigma_{\text{cf}} / \sigma_R$ decreases with increasing bombarding energy
• Neutron deficient heavy ions produce compound nuclei on neutron-deficient side of $\beta$ stability belt
• Heavy ion of energy above Coulomb barrier brings enough excitation energy to evaporate several nucleons
  ▪ 5-10 MeV deexcitation for neutron evaporation
• Heavy-ion reactions needed for reaching predicted island of stability around $Z=114$ to $N=184$
• $U$ is excitation energy, $M_A$ and $M_a$ masses of target and projectile, $T_a$ is projectile kinetic energy, $S_a$ is projectile binding energy in compound nucleus

\[
U = \frac{M_A}{M_A + M_a} T_a + S_a
\]
Photonuclear reactions

- Reactions between nuclei and low- and medium-energy photons dominated by giant resonance
  - Excitation function for photon absorption goes through a broad maximum a few MeV wide
    → Due to excitation of dipole vibrations of protons against neutrons in the nucleus
- Resonance peak varies smoothly with A
  - 24 MeV at $^{16}$O
  - 13 MeV at $^{209}$Bi
- Peak cross sections are 100-300 mb
- ($\gamma$, p), ($\gamma$, n), ($\gamma$,α) reactions

http://www.engin.umich.edu/research/cuos/ResearchGroups/HFS/Research/photonuclear_reactions.html
Natural Element Production

- **Nuclear Astrophysics**
  - fundamental information nuclear properties and reaction
  - properties of astronomical objects

- **Nuclear reactions responsible for production of elements**
  - Occurs in stars

- **At temperatures and densities**
  - light elements have enough thermal velocities to induce nuclear reaction
  - heavier elements created by variety of nuclear processes in massive stellar systems

- systems must explode to disperse the heavy elements

- underlying information on elemental abundances

- nuclear processes to produce primordial elements
Formation of elements

- **Big bang** 15E9 years ago
- **Temperature** 1E9 K
- **Upon cooling** influence of forces felt
  - 2 hours
    - $\rightarrow$ H (89 %) and He (11 %)
  - Free neutrons decay
  - H and He present after quark condensation and initial nuclear reactions
- **Actinides** some distance from stable elements
  - Different reactions formed different elements and isotopes
Origin of Elements

- Gravitational coalescence of H and He into clouds
- Increase in temperature to fusion
- Proton reaction
  - $^1\text{H} + \text{n} \rightarrow ^2\text{H} + \gamma$
  - $^2\text{H} + ^1\text{H} \rightarrow ^3\text{He}$
  - $^2\text{H} + \text{n} \rightarrow ^3\text{H}$
  - $^3\text{H} + ^1\text{H} \rightarrow ^4\text{He} + \gamma$
  - $^3\text{He} + \text{n} \rightarrow ^4\text{He} + \gamma$
  - $^3\text{H} + ^2\text{H} \rightarrow ^4\text{He} + \text{n}$
  - $^2\text{H} + ^2\text{H} \rightarrow ^4\text{He} + \gamma$
  - $^4\text{He} + ^3\text{H} \rightarrow ^7\text{Li} + \gamma$
  - $^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma$
  - $^7\text{Be}$ short lived
  - Initial nucleosynthesis lasted 30 minutes
    - * Consider neutron reaction and free neutron half life
- Further nucleosynthesis in stars
  - No EC process in stars
Stellar Nucleosynthesis

- **He burning**
  - $^4\text{He} + ^4\text{He} \leftrightarrow ^8\text{Be} + \gamma - 91.78 \text{ keV}$
    - Too short lived
  - $3\ ^4\text{He} \rightarrow ^{12}\text{C} + \gamma + 7.367 \text{ MeV}$
  - $^{12}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O}$
  - $^{16}\text{O} + ^4\text{He} \rightarrow ^{20}\text{Ne}$

- **Formation of $^{12}\text{C}$ based on Hoyle state**
  - Excited nuclear state
    - Somewhat different from ground state $^{12}\text{C}$
    - Around 7.6 MeV above ground state
    - $0^+$

- **Fusion up to Fe**
  - From binding energy curve
  - Maximum at Fe
Stellar Nucleosynthesis

- **CNO cycle**
  - $^{12}\text{C} + ^1\text{H} \rightarrow ^{13}\text{N} + ^1\text{H} + \gamma$
  - $^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_e$
  - $^{13}\text{C} + ^1\text{H} \rightarrow ^{14}\text{N} + \gamma$
  - $^{14}\text{N} + ^1\text{H} \rightarrow ^{15}\text{O} + \gamma$
  - $^{15}\text{O} \rightarrow ^{15}\text{N} + e^+ + \nu_e$
  - $^{15}\text{N} + ^1\text{H} \rightarrow ^{12}\text{C} + ^4\text{He}$
  - **Net result is conversion of 4 protons to alpha particle**
    $$4 \ ^1\text{H} \rightarrow ^4\text{He} + 2\ e^+ + 2\ \nu_e + 3\ \gamma$$
  - $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{20}\text{Ne} + ^4\text{He}$
  - $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{23}\text{Na} + p$
  - $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{23}\text{Mg} + n$
  - $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg} + \gamma$
  - $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{24}\text{Mg} + 2\ ^4\text{He}$
  - $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{28}\text{Si} + ^4\text{He}$
  - $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{31}\text{P} + p$
  - $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{31}\text{S} + n$
  - $^{16}\text{O} + ^{16}\text{O} \rightarrow ^{32}\text{S} + \gamma$
Formation of elements $A>60$

Neutron Capture; S-process

- $A>60$
- $^{68}\text{Zn}(n, \gamma) ^{69}\text{Zn}, ^{69}\text{Zn} \rightarrow ^{69}\text{Ga} + \beta^- + \nu$
- Mean times of neutron capture reactions longer than beta decay half-life
  - $\rightarrow$ Isotope can beta decay before another capture
- Up to Bi
Nucleosynthesis: R process

- Neutron capture time scale very much less than $\beta$- decay lifetimes
- Neutron density $10^{28}/\text{m}^3$
  - Extremely high flux
  - capture times of the order of fractions of a second
  - Unstable neutron rich nuclei
- rapidly decay to form stable neutron rich nuclei
- all $A<209$ and peaks at $N=50, 82, 126$ (magic numbers)
P process

- Formation of proton rich nuclei
- Proton capture process
- 70<A<200
- Photonuclear process, at higher Z (around 40)
  - $(\gamma, p)$, $(\gamma, \alpha)$, $(\gamma, n)$
  - $^{190}$Pt and $^{168}$Yb from p process
- Also associated with proton capture process $(p, \gamma)$
- Variation on description in the literature
• Proton-rich nuclei with $Z = 7-26$
  ▪ Forms a small number of nuclei with $A < 100$

• $(p,\gamma)$ and $\beta^+$ decays that populate the p-rich nuclei
  ▪ Also associated with rapid proton capture process

• Initiates as a side chain of the CNO cycle
  ▪ $^{21}\text{Na}$ and $^{19}\text{Ne}$
Review Notes

- Understand Reaction Notation
- Understand Energetics of Nuclear Reactions
  - Q values and barriers
- Understand the Different Reaction Types and Mechanisms
  - Particles
  - Energy
- Relate cross sections to energy
- Describe Photonuclear Reactions
- Routes and reactions in nucleosynthesis
- Influence of reaction rate and particles on nucleosynthesis
Questions

• Describe the different types of nuclear reactions shown on 9-11, lecture 2.

• Provide notations for the following
  ▪ Reaction of $^{12}$C with $^{206}$Pb to make stable Au
  ▪ Formation of Pu from Th and a projectile

• Find the threshold energy for the reaction of $^{59}$Co and an alpha that produces a neutron and a product nuclei

• What are the differences between low and high energy reactions?

$^{206}$Pb($^{12}$C,$^{21}$F)$^{197}$Au

$^{59}$Co($^{4}$He,n)$^{62}$Cu

5.434 MeV
Questions

- How does a charged particle reaction change with energy? A neutron reaction?
- How are actinides made in nucleosynthesis?
- What is the s-process?
- What elements were produced in the big bang?
- Which isotopes are produced by photonuclear reactions?
- What is interesting about the production of $^{12}$C

(γ, p), (γ, n), (γ,α) reactions

Hoyle state and 3 He reaction
Question

• Provide comment in blog
• Respond to PDF Quiz 9