1. For beta decay, what is the correlation between decay energy and half life and how does it differ from the correlation that exists for alpha decay?

There is a qualitative connection between half life and decay energy, although not as simple as case of $\alpha$-decay. There is a strong dependence on spin and parity changes that impact half-life in addition to decay energy. The spin and parity changes involved in beta decay inhibit decay, hence increase the observed half-life.

2. What are the types of decay that are encompassed by beta decay?
Beta decay includes any radioactive decay process in which $A$ remains unchanged, but $Z$ changes with three specific cases: $\beta^-$ decay, electron capture, $\beta^+$ decay

3. What is the basis for the theory of the neutrino emission in beta decay.

- Zero charge
  a. neutron $\rightarrow$ proton + electron
- Small mass
  a. Electron goes up to Q value so the other particle must have small mass
- Anti-particle
  a. Account for creation of electron particle
- Spin of $\frac{1}{2}$ and obeys Fermi statistics
  a. Couple the total final angular momentum to initial spin of $\frac{1}{2}$ $\hbar$

4. In beta decay what are the two possible arrangements of spin?

The spin of the electron and neutrino are $\frac{1}{2}$. These spins can be parallel (spin 1) or antiparallel (spin 0).

5. What are the names of the different spin configurations that can occur during beta decay and where are they dominant?

6. Fermi (F) ($S\beta=0$), Lighter $Z=N$ nuclei, no radical difference in moving neutron to proton shell in shell model.
Gamow-Teller (GT) ($S\beta =1$), Heavier nuclei, likely to encounter spin change when moving neutron to proton in shell model since different states are occupied.

7. Find the Q value for the following. Use mass excess data.

   a. Beta decay of $^{241}$Pu

\[ ^{241}\text{Pu} \rightarrow \beta^- + ^{241}\text{Am} + \bar{\nu} + Q \]

\[ Q=\Delta^{241}\text{Pu} - \Delta^{241}\text{Am} \]

\[ Q=52.952-52.931 \]
b. Positron decay of $^{45}\text{Ti}$

$^{45}\text{Ti} \rightarrow \beta^+ + ^{45}\text{Sc} + \nu + Q$

$Q=\Delta^{45}\text{Ti}-(\Delta^{45}\text{Sc}+2m_e)$

$Q=-39.007\,(-41.070+1.022)$

$Q=1.041$ MeV

c. Electron capture of $^{45}\text{Ti}$

$^{45}\text{Ti} + e^- \rightarrow ^{45}\text{Sc} + \nu + Q$

$Q=m_e+\Delta^{45}\text{Ti}-(\Delta^{45}\text{Sc}+m_e); \text{electron masses cancel}$

$Q=-39.007--41.070$

$Q=2.063$ MeV

d. Beta decay of $^{50}\text{V}$

$^{50}\text{V} \rightarrow ^{50}\text{Cr} + \beta^- + \nu + Q$

$Q=\Delta^{50}\text{V} - \Delta^{50}\text{Cr}$

$Q=-49.220\,-50.258$

$Q=1.038$ MeV

e. Electron capture of $^{138}\text{La}$

$^{138}\text{La} + e^- \rightarrow ^{138}\text{Ba} + \nu + Q$

$Q=\Delta^{138}\text{La} - \Delta^{138}\text{Ba}$

$Q=-86.531\,-88.276$

$Q=1.745$ MeV

f. Positron decay of $^{21}\text{Na}$

$^{21}\text{Na} \rightarrow ^{21}\text{Ne} + \nu + \beta^+ + Q$

$Q=-2.189\,(-5.735+1.022)$

$Q=2.524$ MeV

g. Beta decay of $^{24}\text{Na}$

$^{24}\text{Na} \rightarrow ^{24}\text{Mg} + \beta^- + \nu + Q$

$Q=\Delta^{24}\text{Na} - \Delta^{24}\text{Mg}$

$Q=-8.419\,-13.933$

$Q=5.514$ MeV

8. The decay constant based on Fermi’s golden rule is

$$\lambda_\beta = \frac{2\pi}{\hbar} |M|^2 \rho(E_o)$$

What are the terms in the equations?

$M$ is the matrix element which couples the initial and final states

$\rho$ is the a phase space factor which describes the volume of phase space available for the outgoing leptons and $E_o$ is the density of the states
9. What is the basis for the difference in positron and electron decay as demonstrated below? Coulomb interaction with nucleus accelerates positrons and decelerates electrons.

10. The comparative half-life calculations are:
\[
\log f_{\beta^-} = 4.0 \log E_o + 0.78 + 0.02Z - 0.005(Z - 1) \log E_o
\]
\[
\log f_{\beta^+} = 4.0 \log E_o + 0.79 + 0.007Z - 0.009(Z + 1) \left( \frac{E_o}{3} \right)^2
\]
With \( E_o \) the maximum decay energy (can be the Q value) and \( Z \) the \( Z \) of the daughter. For \( \log f_t \), the half-life is in seconds.

What are the transition types based on these calculations and how is spin related to the types of transition?
Higher spin changes generally increase the forbidden degree of the transition.

11. Use the equations from Question 10 to find the logft values for:

a. Positron decay of $^{45}$Ti

$^{45}$Ti $\rightarrow$ $\beta^+$ + $^{45}$Sc + $\nu$ + Q

Q=1.041 MeV

$T_{1/2}$=3.078 hr

Z=21

Logft=5.00

Actual value 4.59

Allowed transition 7/2- to 7/2-

b. Beta decay of $^{50}$V

$^{50}$V $\rightarrow$ $^{50}$Cr + $\beta^-$ + $\nu$ + Q

Q=1.038 MeV

$T_{1/2}$=1.4E17 years

Z=24

Logft=26.0

Actual value 24 for excited state

Third forbidden transition 6+ to 0+, no parity change

c. Beta decay of $^{24}$Na

$^{24}$Na $\rightarrow$ $^{24}$Mg + $\beta^-$ + $\nu$ + Q

Q=5.514 MeV

$T_{1/2}$=14.95 hours

Z=12

Logft=6.05

1st forbidden transition 4+ to 0+

Decay to excited state decrease spin difference
d. Beta decay of $^{144}\text{Ce}$
\[ Q = 0.318 \text{ MeV} \]
\[ T_{1/2} = 284.6 \text{ days} \]
\[ Z = 59 \]
\[ \log f_t = 7.64 \]
Actual value 7.34
1st forbidden 0+ to 0-. Parity change

12. In electron capture when would the K shell electron capture not be favored?
If the decay is not energetic enough to remove the K shell electron then it will not be favored.